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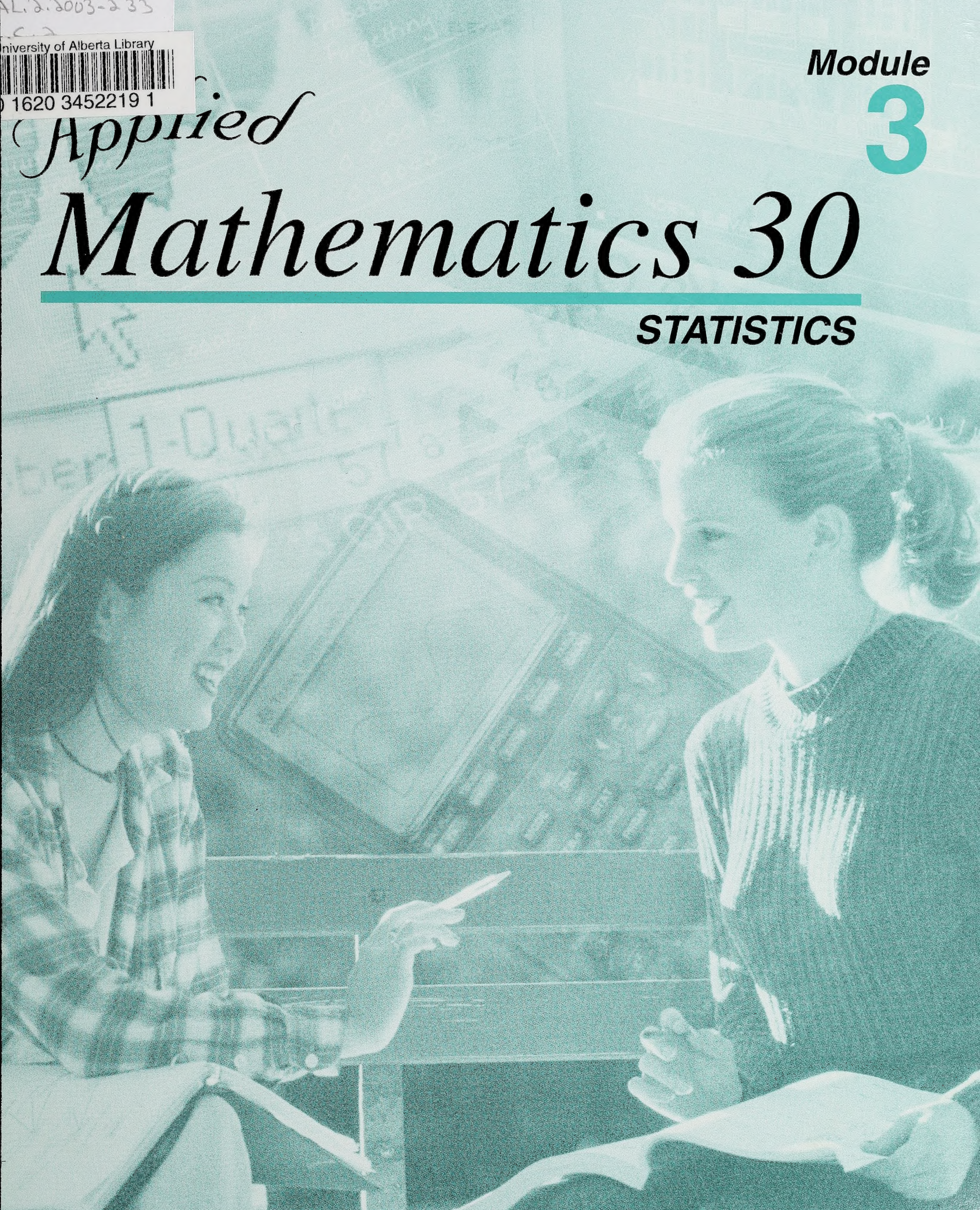
Module

3

Applied Mathematics 30

STATISTICS



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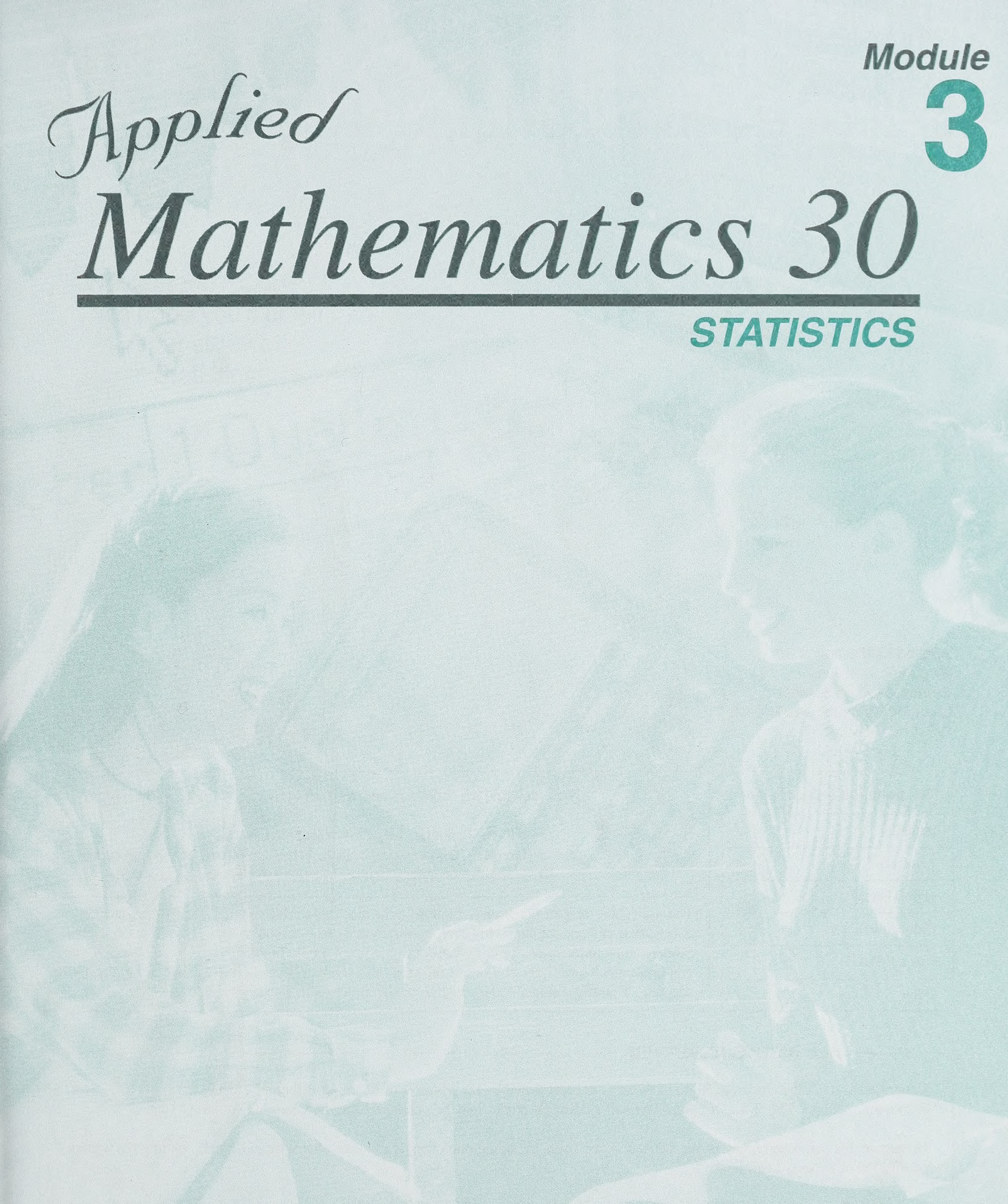
Module
3

Applied Mathematics 30

STATISTICS



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Applied Mathematics 30
Module 3: Statistics
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2197-1

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Learning, **http://www.learning.gov.ab.ca**
- Learning Technologies Branch, **http://www.learning.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.learning.gov.ab.ca**

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

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Welcome

Applied Mathematics 30

- Module 1: Probability
- Module 2: Matrices
- **Module 3: Statistics**
- Module 4: Personal Finance
- Module 5: Sinusoidal Data
- Module 6: Patterns
- Module 7: Vectors

Welcome to Module 3.
We hope you'll enjoy your study of Statistics.

Applied Mathematics 30 contains seven modules and a final test. Work through the modules in the order given, since several concepts build on each other as you progress through the course.

Contents

Introduction to Applied Mathematics 30 ---------- 1

Module Overview
Assessment ---------- 9

Module Project: Opinion Polls
Beginning the Project ---------- 10

Activity 1: Distributions ---------- 12

Activity 2: Mean and Standard Deviation ---------- 21

Activity 3: Normal Distributions ---------- 30

Activity 4: Standard Normal Distribution ---------- 34

Activity 5: The Normal Approximation to a Binomial Distribution ---------- 48

Activity 6: Confidence Intervals ---------- 56

Module Review
Enrichment ---------- 62

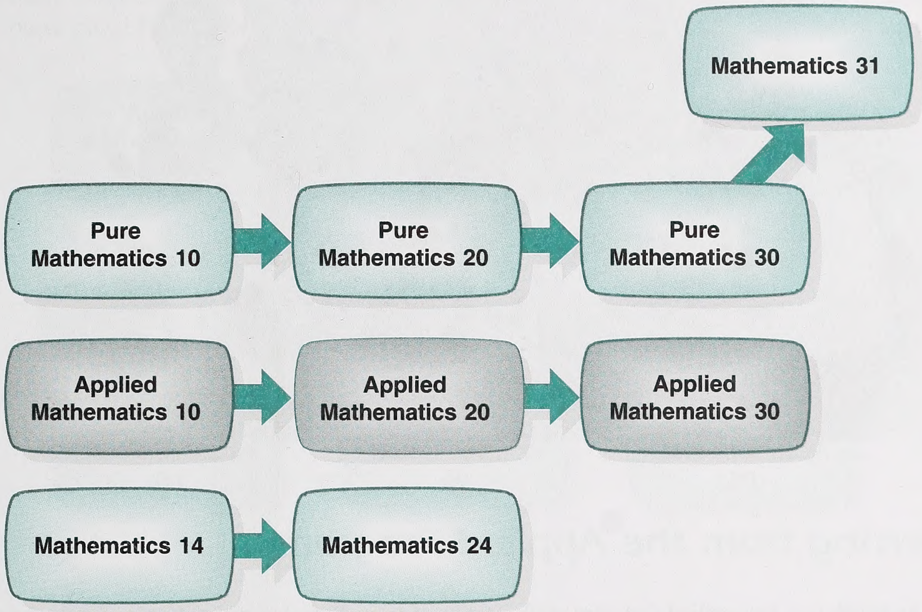
Module Project: Opinion Polls
Completing the Project ---------- 66

Module Summary ---------- 68

Appendix
Glossary ---------- 70
Calculator Functions ---------- 71
Suggested Answers ---------- 74
Image Credits ---------- 135

Introduction to Applied Mathematics 30

Applied Mathematics 30 is the third course in the Applied Mathematics 10–20–30 program of studies. Another program of studies is Pure Mathematics 10–20–30; students who complete Pure Mathematics 30 often choose to take Mathematics 31. A third program of studies is Mathematics 14–24.



Each mathematics program is designed for students with different mathematical strengths and interests.

- Pure Mathematics 10–20–30 is intended for students who are strong in algebra and mathematical theory.
- Applied Mathematics 10–20–30 is better suited to students who prefer to solve problems using numerical reasoning or geometry.
- Mathematics 14–24 is a general mathematics program for high school students who have experienced difficulties in previous mathematics courses.

Each sequence of courses is designed for students with different career plans. For example, Pure Mathematics 30 is a prerequisite for admission to many university programs. Many colleges and technical institutes, however, will admit students who have successfully completed Applied Mathematics 30.

You may find it helpful to read any of the documents under the heading “New Senior High School Mathematics Update/Post-Secondary Studies Update” at the following Internet site:

http://www.learning.gov.ab.ca/k_12/curriculum/bySubject/math

Before enrolling in Applied Mathematics 30, it is recommended that you talk with a school counsellor about your career plans.

Transferring from the Applied Program

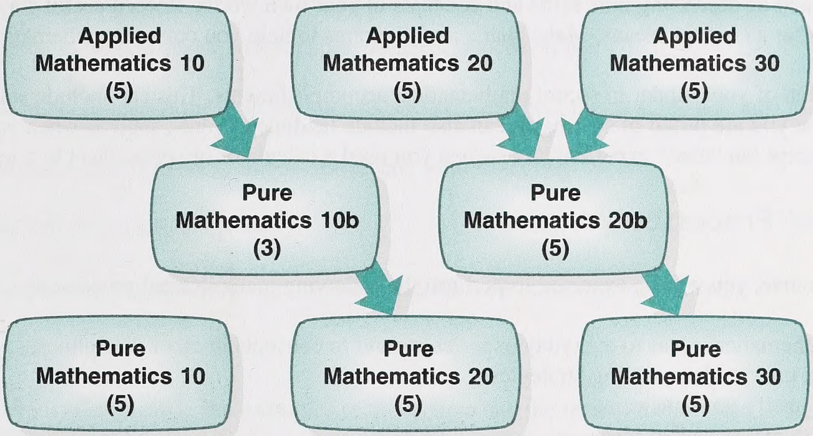
You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are independent.

The following table shows some common and independent topics.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectations • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

If you want to transfer from the Applied Mathematics 10–20–30 sequence to the Pure Mathematics 10–20–30 sequence at a future time, you won't have to repeat the topics that are common to pure mathematics and applied mathematics.

If you decide to transfer to Pure Mathematics 20 after successfully completing Applied Mathematics 10, you may have to take a three-credit course called Pure Mathematics 10b. If you decide to transfer to Pure Mathematics 30 after successfully completing Applied Mathematics 20 or Applied Mathematics 30, you may have to take a five-credit course called Pure Mathematics 20b. The two bridging courses are shown in the following diagram.

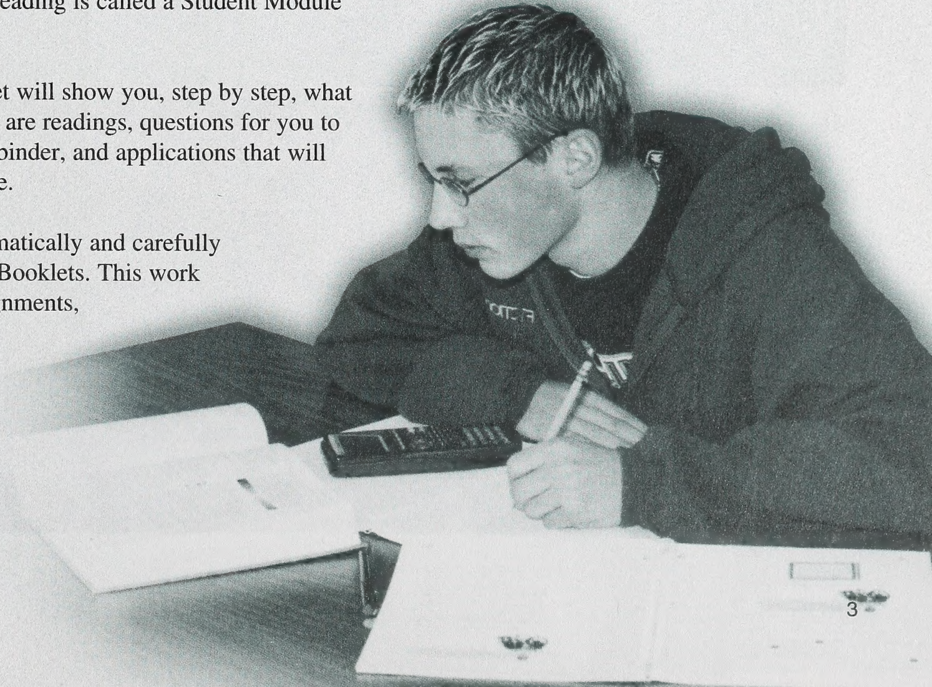


Strategies for Completing Applied Mathematics 30

For each module in Applied Mathematics 30, there is a Student Module Booklet and accompanying Assignment Booklets. The document you are presently reading is called a Student Module Booklet.

Each Student Module Booklet will show you, step by step, what to do and how to do it. There are readings, questions for you to answer in your mathematics binder, and applications that will give you hands-on experience.

It is important to work systematically and carefully through the Student Module Booklets. This work will prepare you for the assignments, projects, and final test.



Following are some suggestions for organizing your mathematics binder:

- Keep a section of your binder to record your responses to the questions in the Student Module Booklet. Also store your marked assignments here.

- Keep a section of your binder for work in progress on your projects. Keep your research notes, plans, rough drafts, and so on.

- Keep a section of your binder to record new skills and concepts, as well as important results and formulas. Get in the habit of describing new skills and concepts in your own words. Record useful ways to help you remember what a concept means. Make charts and diagrams to help you connect mathematical ideas.

- Keep a section of your binder to record mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you grasped a difficult concept (an "aha!" experience), or when you used a calculator or spreadsheet in a new way.

Mathematical Processes

Throughout this course, you will be expected to perform the following mathematical processes:

- Connect mathematical ideas to everyday experiences and to concepts in other disciplines.
- Develop and use problem-solving strategies.
- Reason and justify your answers.
- Communicate mathematical ideas.
- Select and use appropriate technologies to solve problems.
- Develop and use estimation and mental-math strategies.
- Use visualization to assist in processing information, making connections, and solving problems.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Applied Mathematics 30 to be your study partner. You will find that having a friend to discuss mathematical ideas with will make your studying more enjoyable.

Resources You Will Need

In addition to the course materials for Applied Mathematics 30, you will need the following resources:

- the *Addison-Wesley Applied Mathematics 12 Source Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- the *Addison-Wesley Applied Mathematics 12 Project Book*, Western Canadian Edition, published by Addison Wesley Longman Ltd. (2002)
- a binder, lined loose-leaf paper, graph paper, dividers, pencils, eraser
- metric and imperial measuring devices, such as a ruler, yardstick, metre-stick, and tape measure
- a mathematical instrument set (compass, protractor, and triangles)
- a computer with a spreadsheet program

  Note: Two popular spreadsheet programs are *ClarisWorks*™ and Microsoft® *Excel*.
- a graphing calculator

  Note: Where it is applicable, the examples in this course and the textbook show the TI-83 calculator; however, all of the graphing calculators in the following chart are approved for use on tests.

Texas Instruments	Sharp	Casio	Hewlett-Packard
TI-83	EL-9600C	Algebra FX 2.0	HP 39g[†]
TI-83 Plus	EL-9600*	CFX-9850 GA-Plus*	
TI-86	EL-9200*	CFX-9850 G*	
TI-89	EL-9300*	CFX-9800 G*	
TI-92*		FX-9700 series*	
TI-92 Plus			

*no longer commercially available

[†] The HP 39g calculator will remain on the approved list for the 2001–2003 school years and will then be deleted from the approved list.

If you intend to use the TI-83 or TI-83 Plus graphing calculator, it is recommended that you obtain the video program *The TI-83 Graphing Calculator Video Tutor.*

Many of the resources you will need may be purchased locally or from the Learning Resources Centre (LRC). Following is the LRC website:

http://www.lrc.learning.gov.ab.ca

You may wish to discuss the availability of resources with your teacher, as your school division may have a loan policy.

Visual Cues

You will find many visual cues in this course. Colour is used to highlight terms that are defined in the Glossary of the Appendix of each Student Module Booklet. You will also find several icons in the margins. Read the following explanations to discover what the various icons prompt you to do.

Refer to the textbook or the Project Book.

Refer to the Applied Mathematics 30 CD.

Explore the Internet.

Work with a computer.

Contact your teacher for additional information.

Complete specified questions in the Assignment Booklet.

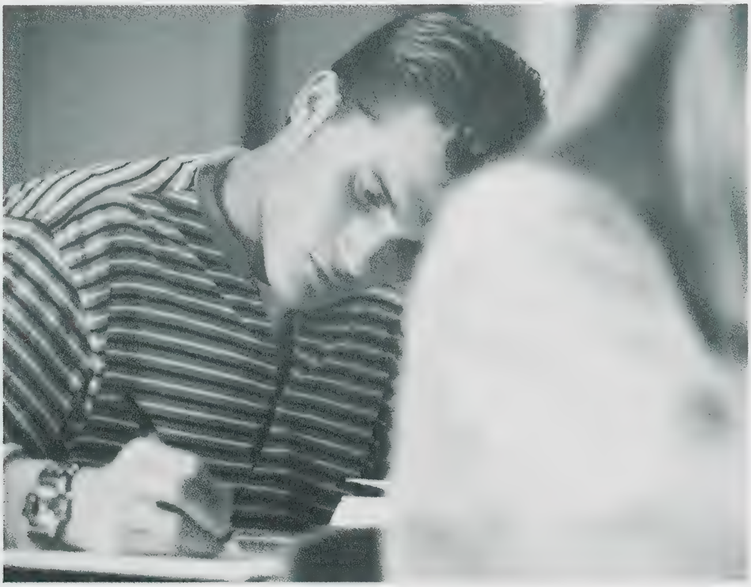
Remember: Any Internet website address given in this module is subject to change.

Where Can I Obtain Diploma Examination Information?

Alberta students will write a diploma examination at the end of the course. Alberta Learning provides several documents to help students prepare for this examination. These documents are found under the heading "Diploma Examinations" at the following Alberta Learning website:

http://www.learning.gov.ab.ca/k_12/testing

Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.



Each year, in February and September, Alberta Learning provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Learning website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions will deal with elements of this project and is worth 10% of your diploma examination mark.

You should take advantage of the many sources of information about Applied Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Applied Mathematics 30.

MODULE OVERVIEW

If you were fortunate enough in 2001 to attend the 8th IAAF World Championships in Athletics in Edmonton or the 2002 Winter Olympics in Salt Lake City, you would have seen some of the world's best athletes. Watching these athletes perform inspires many individuals to do their best, not only in athletics but in all aspects of their lives.

Certainly not everyone will become a world-class athelete, a Nobel Laureate, a chess champion, a concert pianist, or a fashion model. Rare abilities in most endeavours are just that—the exception, not the rule. Almost everyone has drawn pictures, but few will ever achieve the creation of a Mona Lisa.

In this module, you will look at distributions that include such measures as marks on exams, scores on IQ tests, height, and athletic performance. From these distributions you will calculate averages, values that indicate the spread of data within these distributions, probabilities, and proportions. One application you will focus on is an application of distributions to opinion polls and the confidence that the results of these polls reflect general opinion.

Assessment

Accompanying this Student Module Booklet are two Assignment Booklets. Your grading in this module will be based upon the assignments you submit for assessment. The mark distribution is as follows:

Assignment Booklet 3A	
Activities 1 to 3 Assignment	**75 marks**
Assignment Booklet 3B	
Activities 4 to 6 Assignment	**75 marks**
Module Review Assignment	**40 marks**
Module Project	**40 marks**
TOTAL	**230 marks**

Remember that Activities 1 to 6 in this Student Module Booklet will prepare you for completing the module project and the module assignment. You should work through these activities carefully and compare your answers with the suggested answers provided in the Appendix.

The Module Review provides a review of the module and an enrichment activity. You may choose to do some or all of the questions in the Module Review. Again, you should compare your answers with the suggested answers provided in the Appendix.

MODULE PROJECT

Opinion Polls

Beginning the Project

Your teacher may not require you to complete all the projects provided in this Applied Mathematics 30 course. Contact your teacher and check whether you need to complete the module project, Opinion Polls, as part of your assessment.

Have you ever been asked for your opinion on a survey? Governments use opinion polls to gauge the public's mood to help them set policy directions. Political parties survey voters to assess the electorate's mood, particularly during elections. Corporations use survey results to target consumer advertising. School boards survey parents, students, and the community at large to determine the degree of satisfaction with public education.

Some surveys allow respondents a large degree of freedom in responding to general questions. Other surveys limit answers to a selection from a list, to rating categories on a 1 to 5 scale, or even to a simple yes or no to the question posed.

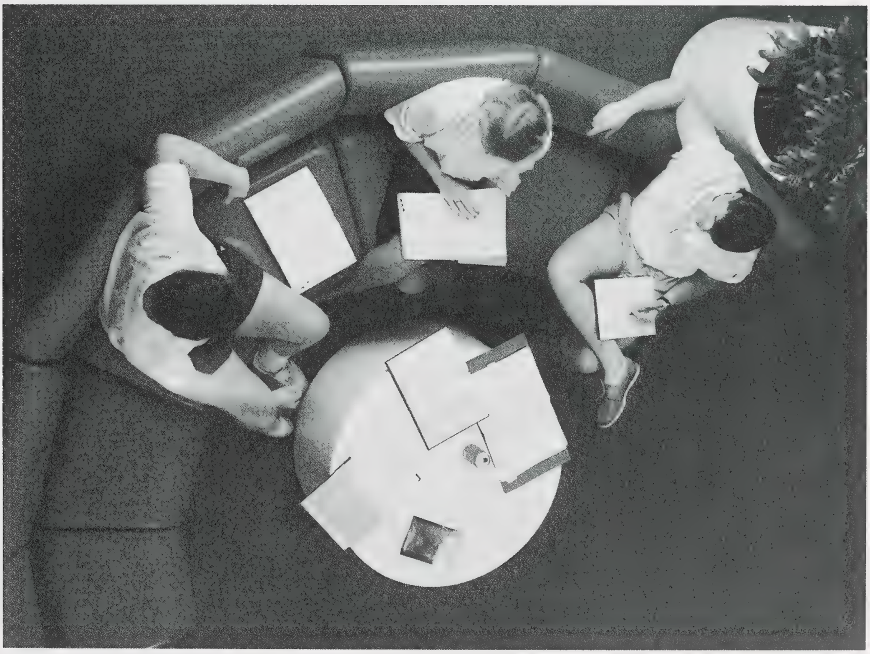
Your project for Module 3 is Opinion Polls. In this project, you will review the media for survey results and discuss how the media reports these surveys. You will draft and conduct your own survey. You will use the data you collect from your survey to produce a report about your results. When you summarize your results, you will include, as part of your analysis, the level of confidence that your survey results accurately represent this population. Check with your teacher to confirm what work on this project you are to submit.

Turn to page 96 of the textbook and read "Opinion Polls." Once you have read the page, complete the activites suggested in the last paragraph. Record the results of your research in the project section of your mathematics binder.

You may wish to begin your research by visiting Addison-Wesley Longman Ltd.'s Internet site, given on page 97 of the textbook. This website provides information you will have to consider when drafting, administering, analysing, and reporting the results of your survey.

As you work through Activities 1 through 6, continue to research the direction your survey will take. You may do this research by reading survey reports in newspapers and magazines, by visiting your school or public library, and by visiting the websites of research firms.

You will be given more direction on how to complete this project later in this module. In the meantime, feel free to discuss your project with your study partner or a family member. Remember, the work on the project you submit must be your own.



ACTIVITY 1

Distributions

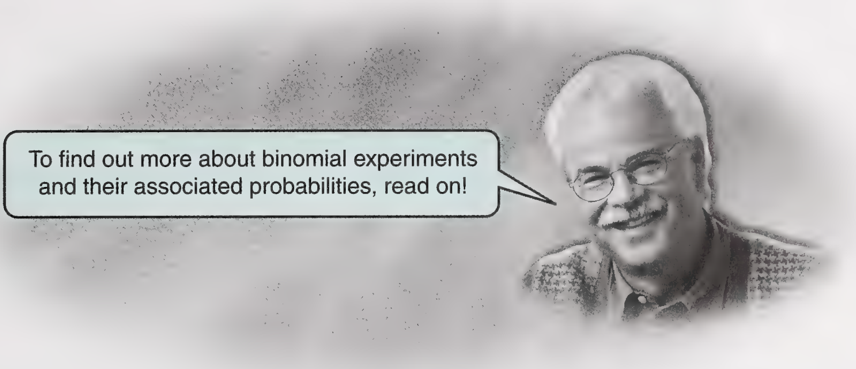
What is the probability of a lacrosse team with a 0.500 record winning three of their next four games in the playoffs against a team with an identical record? What is the probability of a baseball player with a batting average of 0.400 getting exactly one hit at her next three times at bat? What is the probability of getting a mark of 75% on a 20-item, multiple-choice exam by guessing alone?

These questions are all examples of probabilities involving independent events with two possible outcomes. The lacrosse team either wins or loses each game in the playoff round; the baseball player either gets a hit or doesn't get a hit each time at bat; the answer to each multiple-choice question is either right or wrong.

In this activity, you will explore the probabilities arising from experiments, such as those described previously, consisting of a series of trials that are identical, independent, and have only two possible outcomes: success or failure. These experiments or trials are called **binomial experiments**.

Binomial experiments are often referred to as Bernoulli trials, named after Jacob Bernoulli (1654–1705), one of more than a dozen mathematicians from the same family. If you have access to the Internet, you can find out more about Jacob Bernoulli and the Bernoulli family of mathematicians at the following website:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Bernoulli_Jacob.html

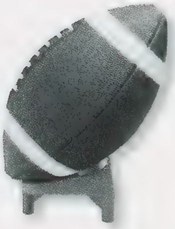

To find out more about binomial experiments and their associated probabilities, read on!

Turn to page 98 of the textbook and read the introductory paragraphs of Tutorial 3.1, "Distributions."

In the reading, you encountered the terms **mean**, **median**, **mode**, and **range**. Recall that these terms are associated with sets of data. The *mean* of a set of data is determined by adding the data values together and then dividing the sum by the number of data values in the set; the *median* of a set of data is the middle value when the data values are ranked from smallest to largest; the *mode* is the most frequently occurring value in a data set; and the *range* is the difference between the largest and smallest values of the data set.

Example

Five rookies are trying out for a defensive-end position on their high school football team. The rookies, ranked from lightest to heaviest, weigh 185 pounds, 190 pounds, 195 pounds, 200 pounds, and 200 pounds.



a. What are the mean, median, mode, and range of the weights?

b. The rookies are joined by three returning players who weigh 192 pounds, 198 pounds, and 210 pounds. What are the mean, median, mode, and range of the weights of all eight players?

Solution

a.

$$\begin{aligned}\text{Mean} &= \frac{185+190+195+200+200}{5}\\ &= \frac{970}{5}\\ &= 194\end{aligned}$$

The mean weight is 194 pounds.

To find the median, you must first rank the weights from lightest to heaviest; then choose the middle value.

185, 190, 195, 200, 200

The middle value is 195. Therefore, the median weight is 195 pounds.

The most frequently occurring weight is 200 pounds. Therefore, the mode is 200 pounds.

$$\begin{aligned}\text{Range} &= \text{Heaviest} - \text{Lightest}\\ &= 200 - 185\\ &= 15\end{aligned}$$

The range is 15 pounds.

b. $$\begin{aligned}\text{Mean} &= \frac{185 + 190 + 195 + 200 + 200 + 192 + 198 + 210}{8}\\ &= \frac{1570}{8}\\ &= 196.25\end{aligned}$$

The mean weight is 196.25 pounds.

The players' weights ranked from lightest to heaviest are as follows:

185, 190, 192, 195, 198, 200, 200, 210

↑ median

The median value lies halfway between the fourth and fifth weights.

$$\begin{aligned}\therefore \text{ Median} &= \frac{195 + 198}{2}\\ &= 196.5 \text{ pounds}\end{aligned}$$

The most frequently occurring weight is still 200 pounds. Therefore, the mode is 200 pounds.

$$\begin{aligned}\text{Range} &= \text{Heaviest} - \text{Lightest}\\ &= 210 - 185\\ &= 25 \text{ pounds}\end{aligned}$$

You will use these terms in your exploration of data distributions.

1. Turn to pages 98 and 99 of the textbook and complete exercises 1 to 6 of "Investigation: Constructing Probability Distributions."

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 74–76.

Turn to page 1 of Assignment Booklet 3A and answer question 1.

2. Turn to page 99 of the textbook and complete exercises 8 to 12 of "Investigation: Constructing Probability Distributions." For exercise 8, use the experimental probability distribution from question 1 of Assignment Booklet 3A.

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 76–79.

In the preceding investigation, you explored tossing a coin four times and recording the number of heads that turned up. This experiment is an example of a binomial experiment. For each toss, the probability of obtaining a head is 50%. Each toss is independent of any other toss, meaning the probability for a given toss is not affected by what has occurred on previous tosses. For each toss there are two possible outcomes: success (obtaining a head) and failure (obtaining a tail). In this experiment, you constructed a **binomial distribution**—the distribution of the probabilities for obtaining 0, 1, 2, 3, or 4 heads on four tosses of a fair coin—using a tree diagram.

You can use your graphing calculator to draw a histogram of the probability distribution and to calculate the probabilities in the distribution. Turn to pages 349 and 350 of the textbook and read "Utility 27: Using the T1-83 to Calculate and Graph a Binomial Distribution."

Study the following example to practise using your calculator to build a histogram of binomial data.

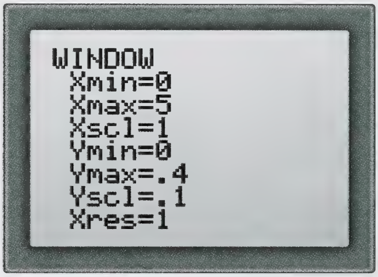
Example

a. Use your graphing calculator to draw a histogram of the probability distribution of obtaining 0, 1, 2, 3, or 4 heads when a fair coin is tossed four times.

b. Use the histogram and the Trace feature of your calculator to obtain the probabilities in the distribution.

Solution

a. Use the window settings given on the right.



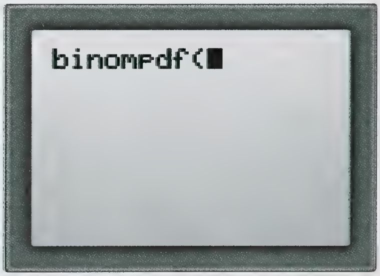
```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=1
 Ymin=0
 Ymax=.4
 Yscl=.1
 Xres=1
```

Remember, you are creating a histogram that represents the probabilities for 0, 1, 2, 3, and 4 heads. That is why you set $\text{Xmin} = 0$. The first bar in the histogram will be the probability of obtaining 0 heads. $\text{Xscl} = 1$ and $\text{Xmax} = 5$ so each bar is 1 unit wide. The bar representing 0 heads will run from $x = 0$ to $x = 1$; the bar representing 1 head will run from $x = 1$ to $x = 2$; and so on to the bar representing 4 heads going from $x = 4$ to $x = 5$.

$\text{Ymax} = 0.4$ because the largest probability determined in the investigation was 0.375 (for 2 heads). However, if you didn't know this, you could always set $\text{Ymax} = 1$, and then adjust it later after viewing your histogram.

Next, you will select the Binomial Probability Distribution feature on your graphing calculator.

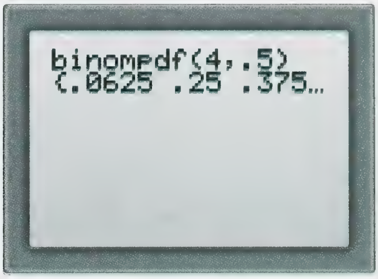
2nd [DISTR] 0 (0:binompdf()



```
binompdf(
```

Because the coin is tossed four times (the number of trials in the experiment) and the probability of getting heads (success) on each toss is 0.5, press the following.

4 , . 5) ENTER

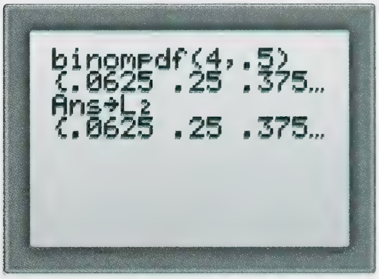


```
binompdf(4,.5)
{.0625 .25 .375…
```

The probability distribution is 0.0625, 0.25, 0.375, 0.25, 0.0625.

To store this distribution so you can graph the histogram later, store it as list L2 by pressing the following.

STO→ 2nd [L2] ENTER



```
binompdf(4,.5)
{.0625 .25 .375…
Ans→L2
{.0625 .25 .375…
```

You should recognize this list as the probabilities in the probability distribution table you completed earlier in "Investigation: Constructing Probability Distributions."

Number of Heads	Probability
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

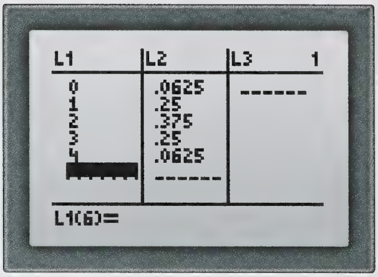
Remember what this table tells you. For example, the probability of obtaining exactly 1 head when a fair coin is tossed four times is 0.25, or 25%.

Now that you have entered these probabilities in list L2, you will need to enter the corresponding number of heads in list L1. Press STAT 1 (1:Edit…) and enter 0, 1, 2, 3, and 4 in L1 as shown.

Note: You may have to clear L_1 first by pressing the following.

Place the cursor over L1.

STAT 1 ▲ CLEAR ENTER

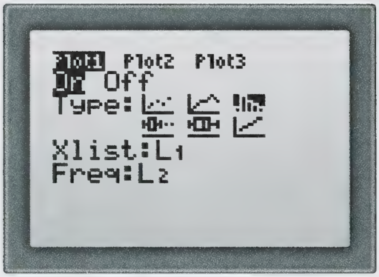


```
L1     |L2     |L3      1
0      |.0625  |------
1      |.25    |
2      |.375   |
3      |.25    |
4      |.0625  |
------ |------ |
L1(6)=
```

Your calculator display is now just like the probability distribution table.

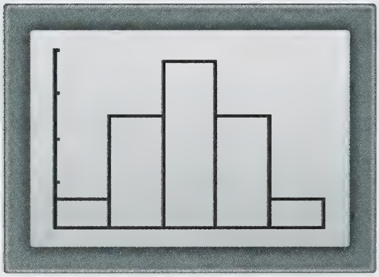
Next, you will graph this table as a histogram. Because a histogram is a statistical graph, you must turn this feature on.

Press 2nd [STAT PLOT] ENTER, and select the following.

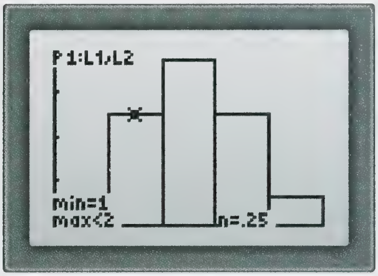


```
Plot1  Plot2  Plot3
On Off
Type:
Xlist:L1
Freq:L2
```

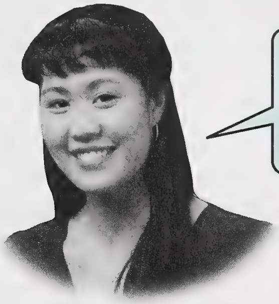
Press GRAPH to display the histogram.



b. Use the Trace feature to review the probabilities.



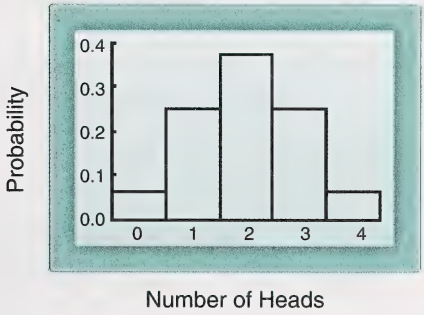
The left- and right-arrow keys can be used to display the probability for any bar. The probability shown here is for 1 head.



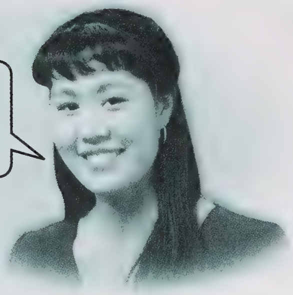
While the calculator display gives the bars of the histogram, you will have to add a title and labels for both the vertical and horizontal axes before the histogram is complete.

The completed histogram looks like the following.

Probability Distribution of Heads
When 4 Coins Are Tossed



Next, you will examine an example of a binomial distribution in which the probability for success for each trial is not 50%.



Turn to page 99 of the textbook and read the material following "Investigation: Constructing Probability Distributions" to the bottom of page 101, working through "Example: Classify Probability Distributions."

3. Answer the following on pages 102 to 104 of the textbook.

   a. exercises 1, 2, and 3 of "Discussing the Ideas"
   b. exercises 2, 4, 6.a., and 6.b. of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 1, pages 79–81.

Turn to page 2 of Assignment Booklet 3A and answer questions 2 and 3.

Looking Back

In this activity you were introduced to probability distributions and histograms. You will explore binomial probability distributions further in this module.

4. Turn to page 104 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 1, page 81.

ACTIVITY 2

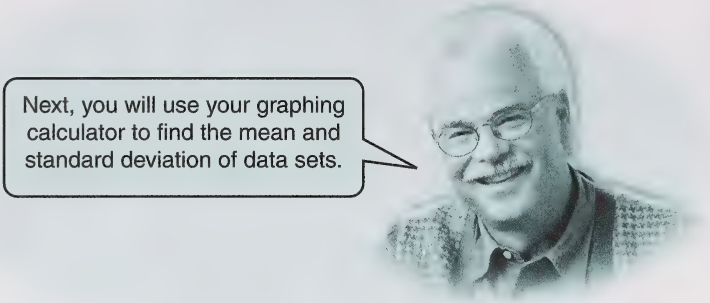
Mean and Standard Deviation

Fashions change over time. If you look at fashions that were current at the turn of the last century, they now look quaint and dated; however, to the people of the day, they were perfectly normal. Look in your own closet. Are there any clothes there you thought were the height of fashion when you bought them, but you simply wouldn't wear today? Fashion experts claim that one of the reasons styles change is that one person, or a small group of influential people, dares to dress in a way that deviates from the norm. Others then copy them.

What is the norm or average? To what degree do items deviate or differ from the average? These are questions that are addressed not only by fashion critics, but also by statisticians.

In this activity, you will explore how to determine the average or mean of a set of data. You will also investigate a measure called **standard deviation**—the extent to which data differs from the mean. A large standard deviation indicates that the data is spread out; a small standard deviation indicates the data is tightly clustered about the mean.

Turn to page 105 of the textbook and read from the introductory paragraphs of Tutorial 3.2, "Mean and Standard Deviation."


Next, you will use your graphing calculator to find the mean and standard deviation of data sets.

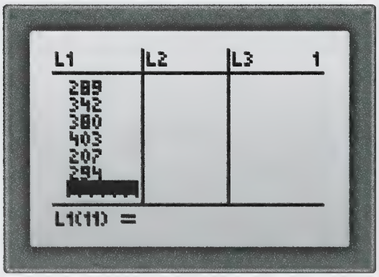
The TI-83 graphing calculator uses the symbol $\overline{x}$ for the mean and σx for the standard deviation. If you are using a different graphing calculator, check your manual. The Greek letter μ (pronounced mu) is often used to represent the mean; it corresponds to the letter *m*—the first letter of the word *mean*. Also, the Greek letter σ (pronounced sigma) corresponds to the letter *s*—the first letter of the term *standard deviation*.

Example

At Alpine Village, the annual snowfall (in cm) for the last 10 years has been 321, 267, 232, 304, 289, 342, 380, 403, 207, and 294. What is the mean and standard deviation? Round your answer to the nearest tenth where necessary.

Solution

Press STAT 1 (1:Edit…), and enter the data into your graphing calculator as a list. Press ENTER after each entry.

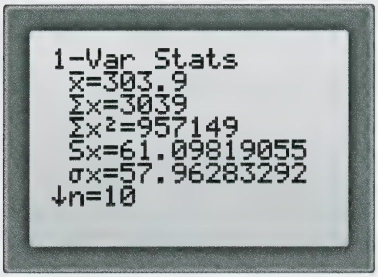


```
L1     |L2     |L3      1
 289   |       |
 342   |       |
 380   |       |
 403   |       |
 207   |       |
 294   |       |
 ------|       |
L1(11) =
```

To determine the mean and standard deviation, press the following:

Select the CALC menu.

STAT ▶ 1 (1:1–Var Stats) ENTER



```
1-Var Stats
 x̄=303.9
 Σx=3039
 Σx²=957149
 Sx=61.09819055
 σx=57.96283292
↓n=10
```

Remember: $\overline{x}$ represents the mean and σx represents the standard deviation.

The mean snowfall is 303.9 cm, and the standard deviation is approximately 58.0 cm.

If a number of data values are repeated, you can find the mean and standard deviation as in the previous example or you can prepare two lists: the first for each distinct data value and the second for the frequency or number of times each data value occurs.

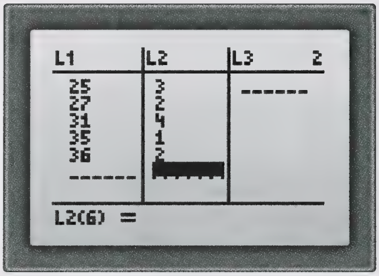
Example

Determine the mean and standard deviation for the following table. Round your answers to the nearest tenth where necessary.

Data Value	Frequency
25	3
27	2
31	4
35	1
36	2

Solution

Enter the data values in list L1 and their corresponding frequencies in list L2. Make sure each data value and its corresponding frequency appear in the same row.



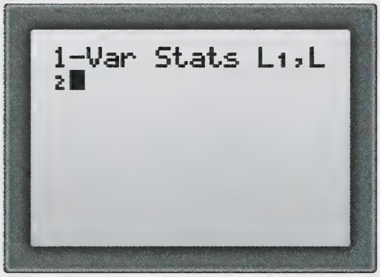
```
L1      L2      L3     2
 25      3      ------
 27      2
 31      4
 35      1
 36      2
------
L2(6) =
```

Remember: To clear an existing list, like the one you made in the preceding example, place the cursor over "L1" and press CLEAR ENTER. Do the same for the other lists.

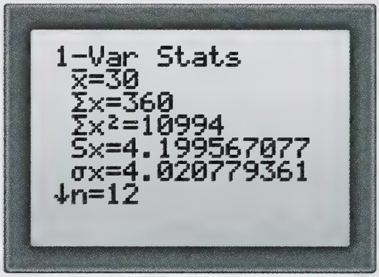
Once you have entered the lists, determine the mean and standard deviation.

Select the CALC menu.

STAT ▶ 1 (1:1–Var Stats) 2nd [L1] , 2nd [L2] ENTER



```
1-Var Stats L1,L
2
```



```
1-Var Stats
 x̄=30
 Σx=360
 Σx²=10994
 Sx=4.199567077
 σx=4.020779361
↓n=12
```

Note: Always check the value of n. It should be the sum of the frequencies. Here, $n = 12$.

The mean is 30, and the standard deviation is approximately 4.0.

In the next investigation, you will explore sets of data that have the same mean but different standard deviations. This investigation will help you answer the question, "What does the standard deviation tell you?"

1. Turn to pages 105 and 106 of the textbook and complete exercises 1 to 4 of "Investigation 1: Compare Sets of Data."

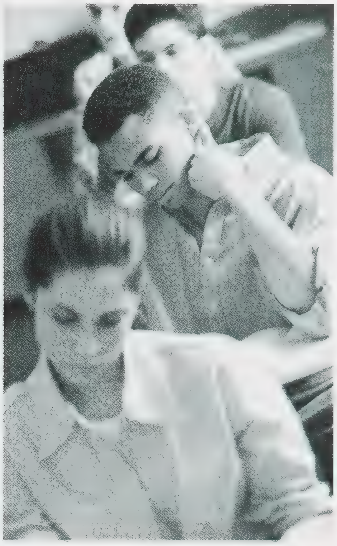
Compare your responses with the suggested answers in the Appendix, Activity 2, pages 81–84.

Turn to page 3 of Assignment Booklet 3A
and answer question 4.

Together, what do the mean and standard deviation of a set of data tell you? The standard deviation is a measure of how the data is clustered about the mean. For large sets of data, approximately 68% of the data lies within one standard deviation of the mean and approximately 95% of the data lies within two standard deviations of the mean. Consider the following example.

Example

On a province-wide Applied Mathematics 30 exam, the mean score was 65 and the standard deviation was 15. Describe the distribution of marks on the exam.



Solution

$\bar{x} = 65$ and $\sigma = 15$

$$\therefore\ \bar{x} - \sigma = 65 - 15 = 50 \quad \text{and} \quad \bar{x} + \sigma = 65 + 15 = 80$$

Approximately 68% of the students who wrote this exam scored between 50 and 80.

$$\begin{aligned} \bar{x} - 2\sigma &= 65 - 2(15) \\ &= 65 - 30 \\ &= 35 \end{aligned} \quad \text{and} \quad \begin{aligned} \bar{x} + 2\sigma &= 65 + 2(15) \\ &= 65 + 30 \\ &= 95 \end{aligned}$$

Approximately 95% of the students who wrote this exam scored between 35 and 95.

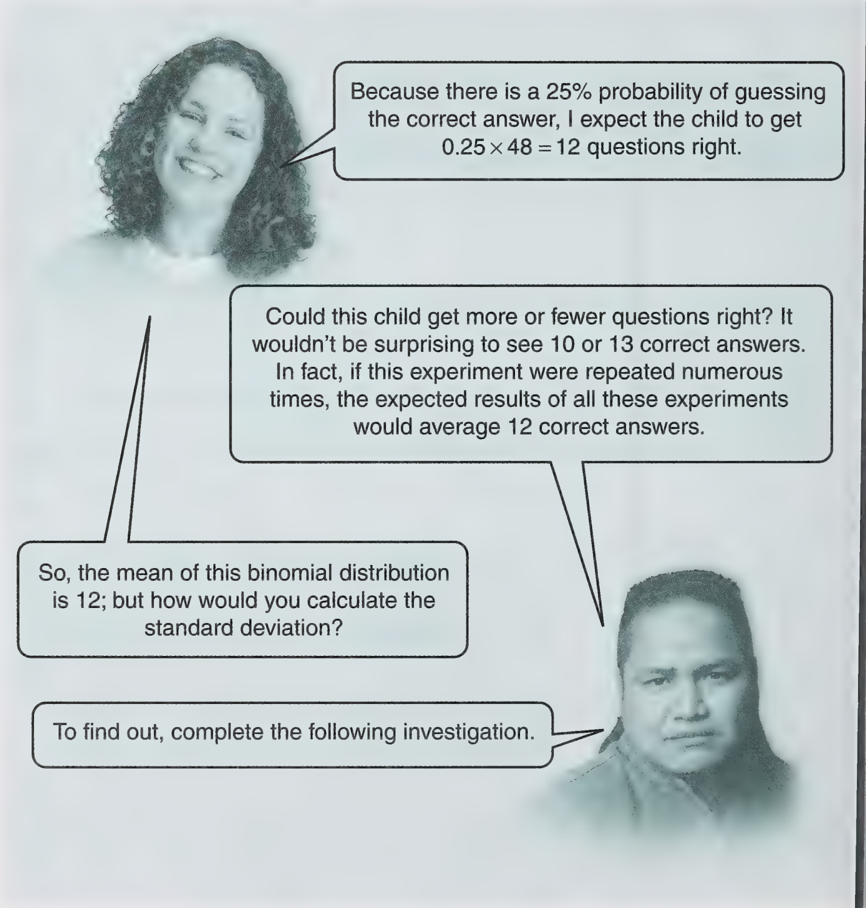
In the next exercise, you will use the concepts of mean and standard deviation to analyse data sets.

2. Turn to pages 108 and 109 of the textbook and answer exercises 1, 3, and 4 of "Exercises: Checking Your Skills."

Compare your responses with the suggested answers in the Appendix, Activity 2, page 84.

In Activity 1 you were introduced to data sets, called binomial distributions. How do you find the mean and standard deviation of a binomial distribution? Consider the following scenario:

> An Applied Mathematics 30 multiple-choice test consisting of 48 items, each with 4 choices, is left on a desk. A young, pre-school child takes a pencil and answers the test by circling one of the 4 choices in each item. How many questions do you think the child would answer correctly?


Because there is a 25% probability of guessing the correct answer, I expect the child to get $0.25 \times 48 = 12$ questions right.
Could this child get more or fewer questions right? It wouldn't be surprising to see 10 or 13 correct answers. In fact, if this experiment were repeated numerous times, the expected results of all these experiments would average 12 correct answers.
So, the mean of this binomial distribution is 12; but how would you calculate the standard deviation?
To find out, complete the following investigation.

3. Turn to pages 106 and 107 of the textbook and complete exercises 1 to 5 of "Investigation 2: Mean and Standard Deviation of a Binomial Distribution." For exercises 3 and 4, consider each to be a trial.

Compare your responses with the suggested answers in the Appendix, Activity 2, pages 85–86.

Turn to page 4 of Assignment Booklet 3A and answer question 5.

In Investigation 2, you discovered the formulas for the mean, μ, and the standard deviation, σ, of a binomial distribution given the number of trials, n, and the probability of success, p, on any given trial.

Mean	Standard Deviation
$\mu = n \times p$	$\sigma = \sqrt{n \times p \times (1-p)}$

Example

A multiple-choice test consisting of 48 items, with 4 choices per item, is answered at random by a large number of students.

a. What would be the mean number of correct answers?
b. What would be the standard deviation of the number of correct answers?
c. Interpret the mean and standard deviation.

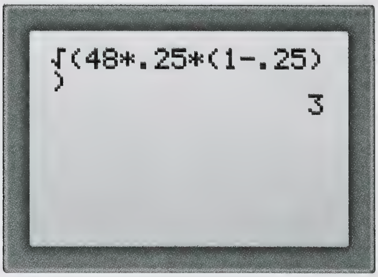
Solution

a. The number of binomial trials, n, is 48; and the probability of success for each trial, p, is 0.25.

$$\begin{aligned} \therefore\ \mu &= n \times p \\ &= 48 \times 0.25 \\ &= 12 \end{aligned}$$

The mean number of correct answers is 12.

b. $$\begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)}\\ &= \sqrt{48 \times 0.25 \times (1-0.25)}\\ &= \sqrt{48 \times 0.25 \times (0.75)}\\ &= 3\end{aligned}$$



```
√(48*.25*(1-.25)
)
                3
```

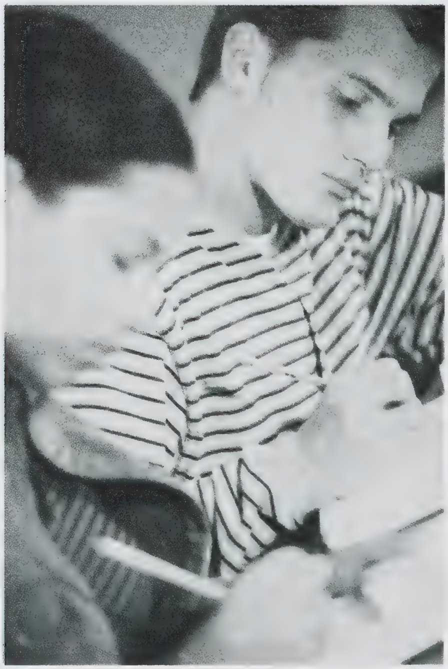
The standard deviation is 3.

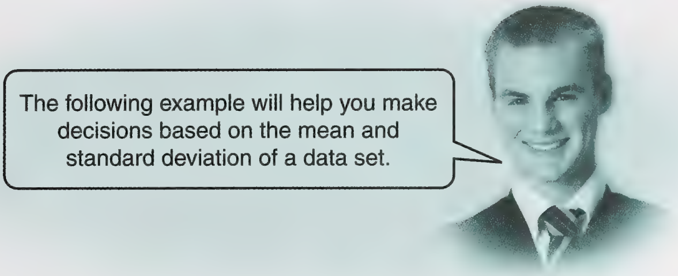
c. $$\begin{aligned}\mu - \sigma &= 12 - 3\\ &= 9\end{aligned} \quad \text{and} \quad \begin{aligned}\mu + \sigma &= 12 + 3\\ &= 15\end{aligned}$$

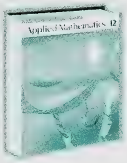
Approximately 68% of the students would answer between 9 and 15 questions correctly on the test.

$$\begin{aligned}\mu - 2\sigma &= 12 - 2(3)\\ &= 12 - 6\\ &= 6\end{aligned} \quad \text{and} \quad \begin{aligned}\mu + 2\sigma &= 12 + 2(3)\\ &= 12 + 6\\ &= 18\end{aligned}$$

Approximately 95% of the students would answer between 6 and 18 questions correctly on the test.




The following example will help you make decisions based on the mean and standard deviation of a data set.



Turn to page 107 of the textbook and work through "Example: Select a Product."

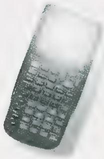
4. Answer the following on pages 108 to 110 of the textbook.

   a. exercises 1, 2, and 3 of "Discussing the Ideas"
   b. exercises 5 and 8 of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 2, page 87.

Turn to page 5 of Assignment Booklet 3A and answer questions 6 and 7.

Looking Back



In this activity you determined the mean and standard deviation of various data sets and binomial distributions using your graphing calculator. You then used these measures to analyse the data sets and distributions.

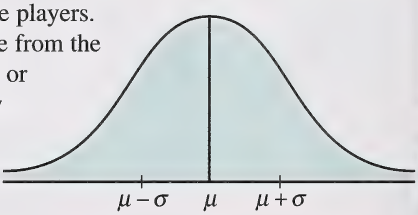
5. Turn to page 110 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 2, page 88.

ACTIVITY 3

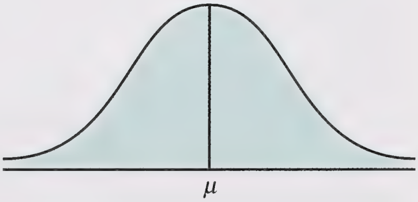
Normal Distributions

Have you heard the term *bell curve* associated with physical and mental attributes, such as height, weight, strength, athletic ability, or intelligence? This term refers to the bell shape of the distribution of these attributes in a population. Take a person's skill level in chess, for example. Most participants are average players. The proportion of players declines as you deviate from the mean, either toward increasing skill on the right, or toward decreasing skill on the left. Think of how many players are in the same category as those who have held the world championship—Boris Spassky, Bobby Fischer, or Garry Kasparov.



Bell-shaped distributions are approximated by the **normal distribution**, defined solely in terms of the mean, μ, and the standard deviation, σ. This function is defined as follows:

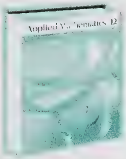
$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$



The graph of this function is called the **normal curve**. You do not have to memorize this function, you just have to know how to use the distribution. Using the normal distribution is the focus of this activity.

The equation of the normal distribution was derived from the binomial distribution by Abraham de Moivre (1667–1754). If you have access to the Internet, you can find out more about Abraham de Moivre at the following website:

http://www-history.mcs.st-andrews.ac.uk/~history/Mathematicians/De_Moivre.html

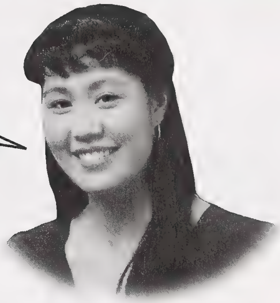


Turn to pages 111 to 113 of the textbook and read the introductory paragraphs of Tutorial 3.3, "The Normal Distribution." Carefully review "Properties of a Normal Distribution" and "68-95-99 Rule" in the coloured boxes. Then work through "Example: Determine Proportions" on pages 113 to 115.

1. Answer the following on page 116 of the textbook.

- **a.** exercises 2 to 5 of "Discussing the Ideas"
- **b.** exercises 1 and 2.a. of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 89–90.



In the next example, you will explore how to use the normal distribution to find probabilities.

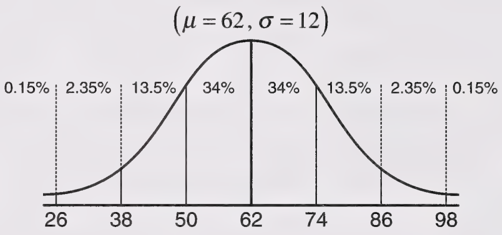
Example

On a province-wide physics test, the mean was 62 and the standard deviation was 12. What is the probability that a randomly selected student who wrote this test will have the following?

- **a.** a mark greater than 62
- **b.** a mark greater than 50
- **c.** a mark less than 86

Solution

Draw a sketch to help visualize the problem.



a. Because 62 is the mean of the distribution, half the students will have marks greater than 62. Therefore, the probability that a randomly selected student who wrote this test will have a mark greater than 62 is 0.5, or 50%.

b. About 34% of the students had marks between 50 and 62, and 50% of the students had marks greater than 62.

$\therefore\ 34\% + 50\% \doteq 84\%$

The probability that a randomly selected student who wrote this test will have a mark greater than 50 is about 0.84, or 84%.

c. About $34\% + 13.5\% = 47.5\%$ had marks between 62 and 86, and 50% of the students had marks less than 62.

$\therefore\ 47.5\% + 50\% \doteq 97.5\%$

The probability that a randomly selected student who wrote this test will have a mark less than 86 is about 0.975, or 97.5%.

2. Turn to pages 117 and 118 of the textbook and answer exercises 3, 5, 6, 7, and 8 of "Exercises: Checking Your Skills." Draw diagrams to help visualize the problems.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 90–92.

Turn to pages 6 and 7 of Assignment Booklet 3A and answer questions 8, 9, and 10.

View the segment *Optimal Design* on the Applied Mathematics 30 CD. This video introduces the idea of reducing cost by careful design.

Project: Un-Designer Jeans

Blue jeans have been a part of most wardrobes for a long time. The first jeans were made out of canvas and sold to miners in the California Gold Rush. They have been changed, improved, and made fashionable through the intervening century and a half. You might find interesting information on jeans at the following Internet sites:

- **http://www.levistrauss.com/about/history/jeans.htm**
- **http://www2.gol.com/users/bobkeim/Garment/jhistory.html**
- **http://web.mit.edu/invent/www/inventorsR-Z/strauss.html**

Turn to pages 46 to 48 of the Project Book and read "The Task" and "Background" of the project titled "Un-Designer Jeans." Answer the questions posed in the bulleted list, and store your responses in your mathematics binder.

For this project, you are going to develop a costing model for the production of your un-designer jeans and design a unique label to apply to each pair of jeans (at least for the first hundred thousand pairs or so).

3. Answer questions 1, 2, and 3.a. of "Getting Started" on page 48 of the Project Book.

Compare your responses with the suggested answers in the Appendix, Activity 3, pages 92–93.

Turn to pages 7 to 11 of Assignment Booklet 3A and answer questions 11 to 17.

Looking Back

In this activity you explored normal distributions and how to apply them to solve problems in real-world situations. You also completed a project that involved developing a costing model for the production of a pair of jeans and designing a unique label.

4. Turn to page 118 of the textbook and answer "Communicating the Ideas."

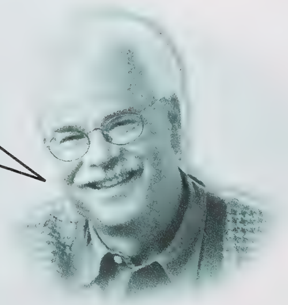
Compare your response with the suggested answer in the Appendix, Activity 3, page 93.

ACTIVITY 4

Standard Normal Distribution

Jasmine has just received her results from two province-wide tests. Her mark in biology is 75 and her mark in physics is 72. Compared to the other students who wrote this exam, on which test did she do better?

To answer this question, you must have a standard of comparison. You need to know the mean and standard deviation of each test.



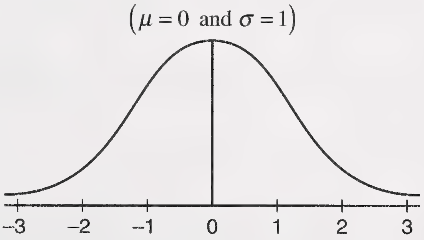
Suppose the mean is 65 and the standard deviation is 10 for the biology test; and suppose the mean is 61 and the standard deviation is 11 for the physics test. Do you agree that each mark is 1 standard deviation above the mean? If the marks on both tests are normally distributed, Jasmine's performance on both tests is the same. Each mark is higher than about 84% of the students on each test, even though her scores on the tests are different.

To make comparisons between different normal distributions, it is often convenient to convert the scores to the number of standard deviations in which they lie above or below the mean. A score of 1 would indicate one standard deviation above the mean; a score of -2 would indicate two standard deviations below the mean; and a score of 0 would indicate the score is at the mean, neither above nor below.

When scores are converted in this way, the mean of the resulting normal distribution is 0 and the standard deviation is 1. This distribution is called a **standard normal distribution**. In this activity, you will explore the standard normal distribution; you will use your graphing calculator or prepared tables to assist you in your analysis.

Turn to page 119 of the textbook and read the introductory paragraphs of Tutorial 3.4, "Standard Normal Distribution" up to and including the coloured box on page 120.

As you have just discovered, the standard normal distribution is obtained from a given standard distribution by converting its scores to the number of standard deviations in which they lie from the mean. The distribution of these converted scores has a mean, μ, of 0 and a standard deviation, σ, of 1.



The converted scores are called **z-scores**, and the formula you will use to obtain these z-scores is as follows:

$$z = \frac{x - \mu}{\sigma}$$

The next example will show you how to convert scores of a given normal distribution into the z-scores of the standard normal distribution.

Example

The heights of male residents in a western Canadian city are normally distributed with a mean of 175.25 cm and a standard deviation of 7.10 cm. Convert the following heights to *z*-scores. Round your answers to the nearest hundredth.

a. 180 cm
b. 168 cm

Solution

a. $\mu = 175.25$ and $\sigma = 7.10$.

$$\text{If } x = 180, \text{ then } z = \frac{x-\mu}{\sigma}$$
$$= \frac{180-175.25}{7.10}$$
$$= \frac{4.75}{7.10}$$
$$\doteq 0.67$$

A height of 180 cm is approximately 0.67 standard deviations above the mean.

b. $$\text{If } x = 168, \text{ then } z = \frac{x-\mu}{\sigma}$$
$$= \frac{168-175.25}{7.10}$$
$$= \frac{-7.25}{7.10}$$
$$\doteq -1.02$$

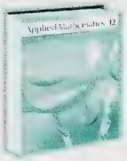
A height of 168 cm is approximately 1.02 standard deviations below the mean.

1. Turn to pages 123 and 124 of the textbook and answer the following.

- **a.** exercises 1 and 2 of "Discussing the Ideas"
- **b.** exercise 1 of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 4, page 94.

Once you have calculated *z*-scores, you can use them to determine probabilities.



Turn to page 120 of the textbook and read the text starting after the coloured box up to Example 1 on page 121.

In the reading, you used a graphing calculator to determine the probability that a randomly selected bag of mini-eggs would have a mass less than 43 g. This probability, or proportion of the distribution, is the area under the standard normal curve lying to the left of the *z*-score of -1.13. If you do not choose to use your graphing calculator, or if you have a graphing calculator that does not have this feature, you can use the *z*-score tables provided on pages 355 and 356 of the textbook.

Example

Use the *z*-score tables to find the area under the standard normal curve lying to the left of a *z*-score of -1.13.

Solution

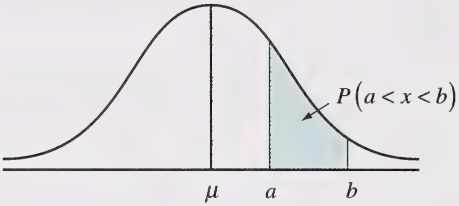
Read along the row headed by -1.1 until you reach the column headed by 0.03.

z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539

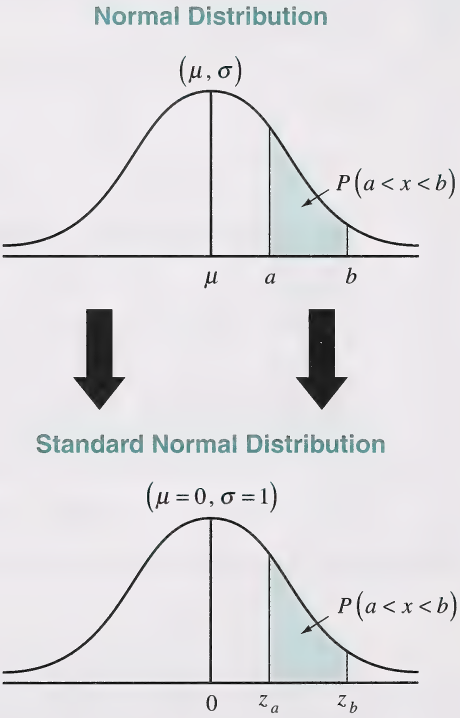
The area lying under the standard normal curve to the left of $z = -1.13$ is 0.1292.

The answer obtained from the tables is the same as the answer obtained using a graphing calculator. Occasionally, because of rounding, the answers may differ in the last digit.

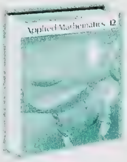
You now know how to use your calculator and the tables to find the proportion of a distribution (the area under a normal curve) lying to the left of a given value in the distribution. How can you determine the area, or proportion, below a normal curve between two values, $x = a$ and $x = b$?



The simplest method, and the one you are encouraged to use, is using the normal distribution feature of your graphing calculator. However, if you choose not to use your calculator, you can use the tables in the textbook (pages 355 and 356). Whether you are using your graphing calculator or the tables, you should first transform the normal distribution you are working with into the standard normal distribution by converting to *z*-scores.



Example

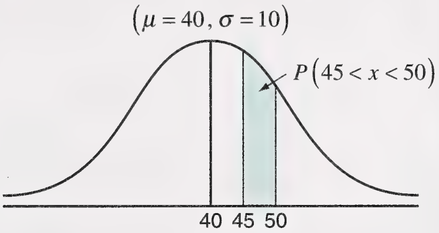


Find the area (or percent or proportion of data) between $x = 45$ and $x = 50$ below a normal curve with a mean of 40 and a standard deviation of 10 using the following.

a. your graphing calculator
b. the tables on pages 355 and 356 of the textbook

Solution

Sketch the distribution before determining the area.



a. Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 45$ and $x = 50$ into z-scores.

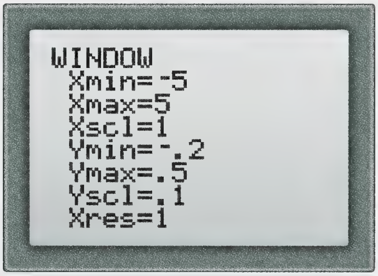
For $x = 45$,

$$z = \frac{x-\mu}{\sigma}$$
$$z_{45} = \frac{45-40}{10}$$
$$= 0.50$$

For $x = 50$,

$$z = \frac{x-\mu}{\sigma}$$
$$z_{50} = \frac{50-40}{10}$$
$$= 1.00$$

Now, you will use your graphing calculator to determine areas under the standard normal curve between these z-scores using the following window settings.



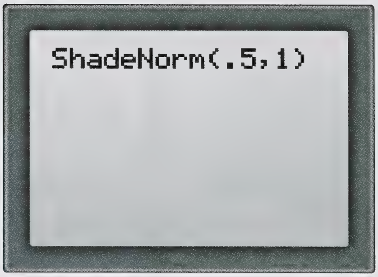
```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-.2
 Ymax=.5
 Yscl=.1
 Xres=1
```

Access the Shade Normal feature by pressing the following.

Select the DRAW menu.

2nd [DISTR] ▶ 1

(1:ShadeNorm()

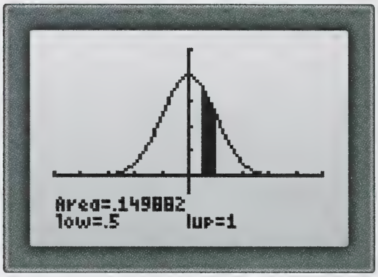


```
ShadeNorm(.5,1)
```

Next, enter the lower and upper bounds of the desired area. (These are the *z*-scores calculated earlier.)

• 5 , 1)

Press ENTER to display the distribution.



```
Area=.149882
low=.5        up=1
```

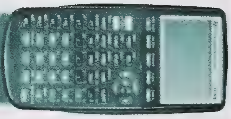
The area below the normal curve between $x = 45$ and $x = 50$ is about 0.1499.

You can express this probability (or proportion) in two ways:

- in terms of the original scores: $P(45 < x < 50) \doteq 0.1499$
- in terms of the z-scores: $P(0.50 < z < 1.00) \doteq 0.1499$

Once you are finished using the Shaded Normal feature on your graphing calculator, you must clear the screen before you draw another distribution; otherwise, the old drawing will appear with your new drawing. To erase the drawing, press the following.

2nd [DRAW] 1 (1:ClrDraw) ENTER



b. From part a., the z-scores of $x = 45$ and $x = 50$ are $z_{45} = 0.50$ and $z_{50} = 1.00$.

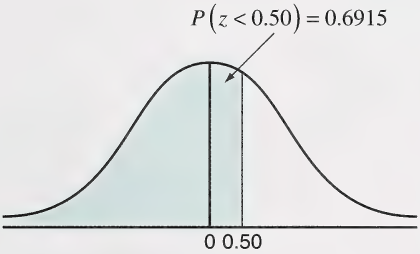
Look up each of these z-scores in the tables on page 356 of the textbook. The values given in the tables are the areas lying under the standard normal curve to the left of these z-scores.

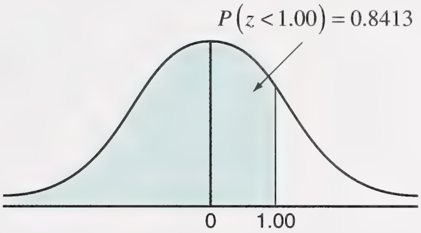
For $z_{45} = 0.50$,

$$P(z < 0.50) = 0.6915$$

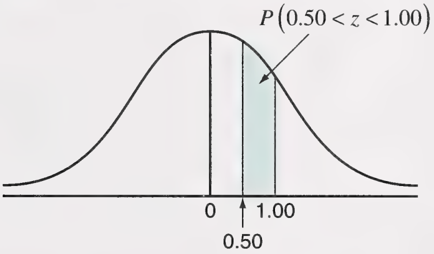
For $z_{50} = 1.00$,

$$P(z < 1.00) = 0.8413$$





To find the area between $z = 0.50$ and $z = 1.00$, subtract the probabilities.

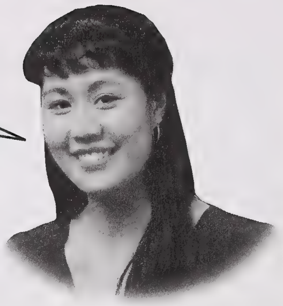


$$\therefore P(0.50 < z < 1.00) = 0.8413 - 0.6915$$
$$= 0.1498$$

The area below the normal curve between $x = 45$ and $x = 50$ is about 0.1498.

Remember: Slight differences between your calculator and the tables are due to rounding. The values in the tables have been rounded to four decimal places.

In the next example, you will see how to find the area under the standard normal curve lying to the right of a given *z*-score.



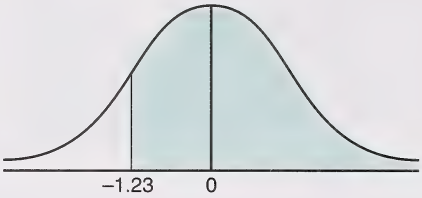
Example

Determine $P(z > -1.23)$ using the following.

a. your graphing calculator
b. the *z*-score tables

Solution

Sketch the distribution.



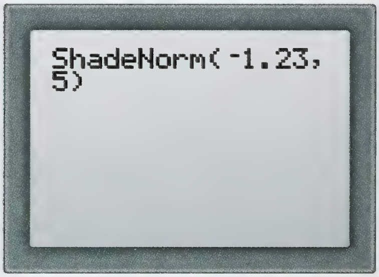
a. Since the *z*-score is negative, it lies to the left of the mean.

Using the same window settings as in the preceding example, use the Shade Normal feature to determine the area.

Select the DRAW menu.

2nd [DISTR] ▶ 1

(1:ShadeNorm()

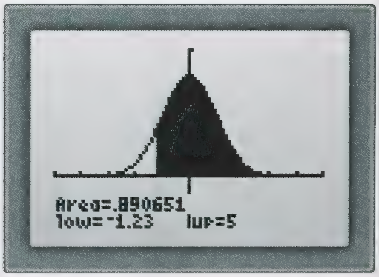


```
ShadeNorm(-1.23,
5)
```

Now, enter the upper and lower bounds (*z*-scores). Use 5 as the upper bound. Only a negligible amount lies to the right of 5.

(–) 1 • 2 3 ,
5)

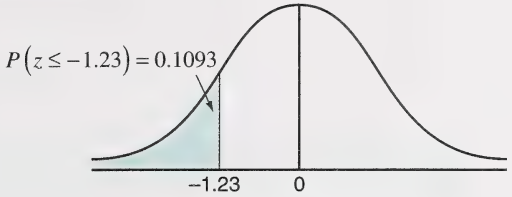
Press ENTER to display the distribution.

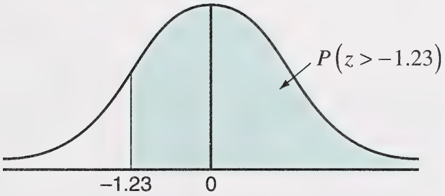


```
Area=.890651
low=-1.23    up=5
```

$\therefore\ P(z > -1.23) \doteq 0.8907$

b. $P(z \leq -1.23) = 0.1093$





Because the entire area under the standard normal curve is 1,

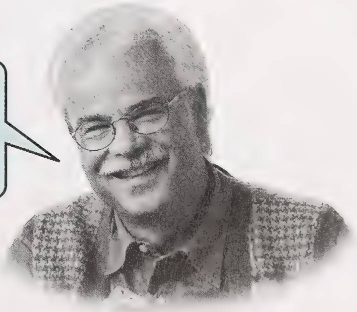
$$\begin{aligned} P(z > -1.23) &= 1 - P(z \leq -1.23) \\ &= 1 - 0.1093 \\ &= 0.8907 \end{aligned}$$

2. Use your graphing calculator to find the area (percent of data) that satisfies the stated condition for each normal distribution. Round your answers to 4 decimal places. Check your answers using the tables in the textbook.

a. between 45 and 85, where the mean is 65 and the standard deviation is 12
b. greater than 67, where the mean is 60 and the standard deviation is 15
c. less than 70, where the mean is 63 and the standard deviation is 14

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 94–98.

Now that you can find areas, proportions, and probabilities using *z*-scores and the standard normal distribution, you will apply these skills to solve a variety of practical problems.



Turn to pages 121 to 123 of the textbook and work through "Example 1: Determine the Number of Ball Bearings that Are Expected to Fail" and "Example 2: Compare Data."

3. Answer the following on pages 123 to 125 of the textbook.

- **a.** exercise 3 of "Discussing the Ideas"
- **b.** exercises 3, 5, 6.a., 7, and 9 of "Exercises: Checking Your Skills"

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 98–105.

Turn to pages 1 to 4 of Assignment Booklet 3B and answer questions 1 to 5.

In all of the previous problems, you used *z*-scores to find areas, proportions, and probabilities. In the next part of this activity, you will work backward from areas, proportions, and probabilities to determine *z*-scores. Then you will use these *z*-scores to answer questions about the original distributions.

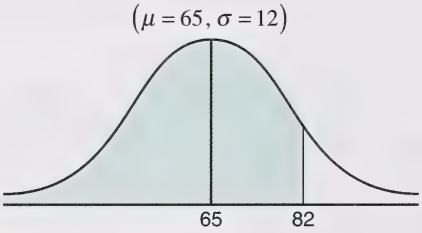
Example

The results on a province-wide mathematics exam are normally distributed with a mean of 65 and a standard deviation of 12.

- **a.** Mary's mark on the exam is 82. What is her percentile ranking?
- **b.** Yerofrei's mark is at the 80th percentile. What was his mark on the exam?

Solution

a. Sketch the distribution.



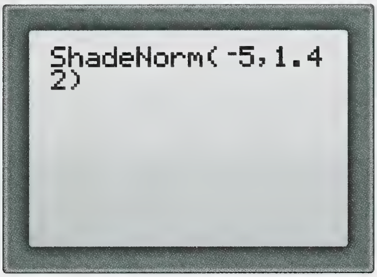
To determine Mary's percentile ranking, you must determine the proportion of students who received a mark on the exam that was less than Mary's. This proportio is the area below the normal curve and less than 82.

First, convert 82 to a z-score.

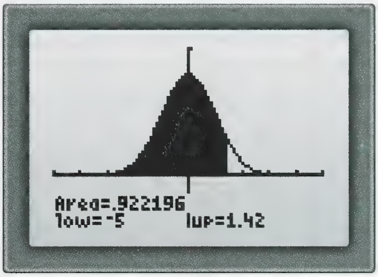
$$x = 82,\ \mu = 65,\ \text{and}\ \sigma = 12$$

$$\begin{aligned} \therefore\ z &= \frac{x-\mu}{\sigma} \\ z_{82} &= \frac{82-65}{12} \\ &\doteq 1.42 \end{aligned}$$

Use your graphing calculator or the z-score tables to determine the area.



```
ShadeNorm(⁻5,1.4
2)
```



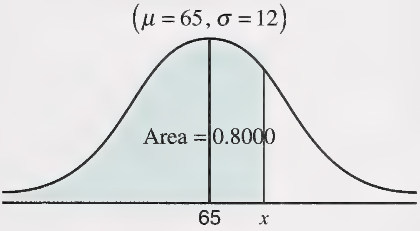
```
Area=.922196
low=⁻5        up=1.42
```

$$\therefore\ P(z < 1.42) \doteq 0.9222$$

Since a mark of 82 is better than approximately 92% of the distribution, Mary's mark of 82 is at the 92nd percentile.

This may be written as $82 = P_{92}$.

b. Sketch the distribution.



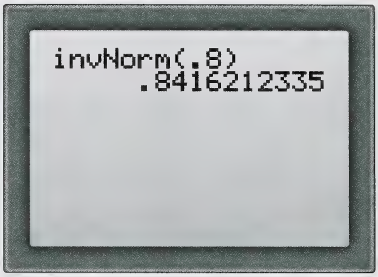
In this example, you must find the mark, x, that corresponds to the 80th percentile. This mark is better than 80% of the distribution. Therefore, the area to the left of the required value is 0.8000. You must work backwards from the area to find a z-score.

Method 1: Using a Graphing Calculator

Use the Inverse Normal Distribution feature on your graphing calculator by pressing the following.

2nd [DISTR] 3 (3:invNorm()

• 8) ENTER

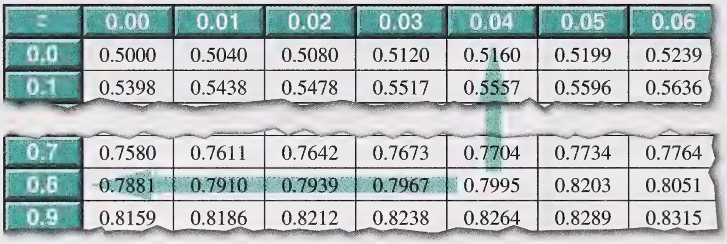


```
invNorm(.8)
      .8416212335
```

The corresponding z-score for which 80% of the distribution lies to the left is approximately 0.84.

Method 2: Using Tables

Look for an area closest to 0.8000 in the z-score tables.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8203	0.8051
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315

According to the tables, the closest value is 0.7995, which corresponds to a z-score of 0.84.

Next, find x using $z = \frac{x-\mu}{\sigma}$.

$z = 0.84$, $\mu = 65$, and $\sigma = 12$

$$
\begin{aligned}
\therefore\ z &= \frac{x-\mu}{\sigma} \\
0.84 &\doteq \frac{x-65}{12} \\
0.84(12) &\doteq x-65 \\
x-65 &\doteq 10.08 \\
x &\doteq 10.08+65 \\
&\doteq 75.08
\end{aligned}
$$

Yerofrei’s mark on the exam was approximately 75.

4. In the general population, the IQ scores are normally distributed with a mean of 100 and a standard deviation of 15. What IQ is necessary to be in the top 10% of the population? Round your answer to the nearest whole number.

5. A shipment of oranges is checked for weight. Approximately 70% of the oranges are smaller than 200 g. Assuming a normal distribution of weights, if the mean weight is 180 g, what is the standard deviation rounded to the nearest tenth? Use the z-score tables.

6. Djon wrote a province-wide social studies exam. The marks were normally distributed. The mean on the test was 63 with a standard deviation of 11. What was Djon's mark if 35% of the students obtained a higher mark? Round your answer to the nearest whole number.

7. A computer manufacturer knows from past experience that the average length of time its product operates without needing repair is 18 months with a standard deviation of 3 months. Assuming a normal distribution, how long should the warranty on its computers be if it does not want more than 5% returned for repair?

Compare your responses with the suggested answers in the Appendix, Activity 4, pages 105–109.

Turn to pages 4 and 5 of Assignment Booklet 3B and answer questions 6 and 7.

Looking Back

In this activity, you explored the standard normal distribution. You used the z-score formula, z-score tables, and your graphing calculator to solve a variety of problems. These problems involved areas, proportions, and probabilities.

You were shown, in the examples and in the Appendix solutions, that you should do the following when solving normal distribution problems:

- sketch the normal curve with given values marked on it
- change values to z-scores
- use the Shade Normal feature to find the required areas

8. Turn to page 125 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 4, page 109.

ACTIVITY 5

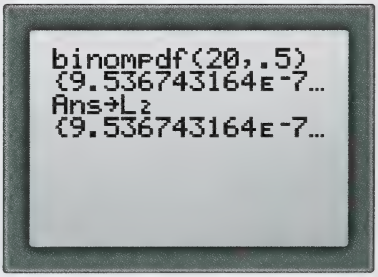
The Normal Approximation to a Binomial Distribution

Flip a toonie 20 times and record the number of heads. How many heads did you obtain? Because the probability of obtaining a head on a single toss is 0.5, you would expect to get 10 heads. However, it is possible to obtain any number of heads from 0 through 20.

Recall, from Activity 1 of this section, that flipping a coin is an example of a binomial experiment. Using your graphing calculator, you can graph the probability distribution of this binomial experiment as a histogram.

Enter the set of whole numbers (0, 1, 2, …, 20) as list L1 and the binomial probability distribution as list L2. For list L2, press the following

2nd [DISTR] 0 (0:binompdf()
2 0 , • 5) ENTER
STO➧ 2nd [L2] ENTER

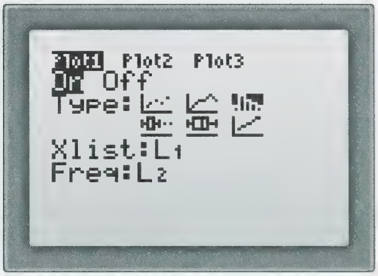


```
binompdf(20,.5)
{9.536743164E-7…
Ans→L2
{9.536743164E-7…
```

To graph the histogram, press

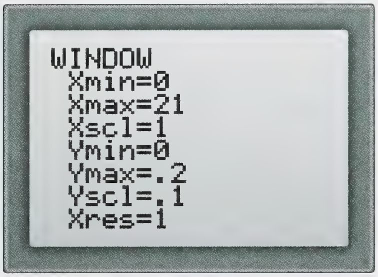
2nd [STAT PLOT] ENTER

Adjust the display to look like the one on the right.

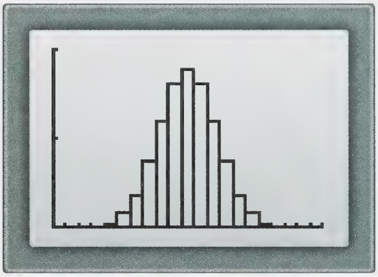


```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```

Using the window settings given, press GRAPH to display the histogram.



```
WINDOW
  Xmin=0
  Xmax=21
  Xscl=1
  Ymin=0
  Ymax=.2
  Yscl=.1
  Xres=1
```

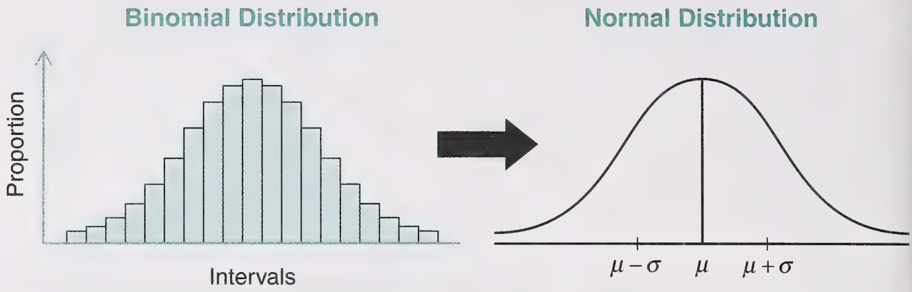


This histogram approximates the normal distribution. The mean of the normal distribution is $20(0.5)=10$, which is the number of heads you would expect in 20 tosses with the probability of obtaining a head on any toss being 0.5.

What is the standard deviation of the normal distribution? How can you use the normal distribution to answer questions about the binomial experiment? What is the probability of obtaining at least 8 heads when a fair coin is tossed 20 times?

You will be able to answer these questions when you complete this activity!

A binomial distribution arising from a binomial experiment consisting of n trials, where the probability of success for each trial is p and the probability of failure of each trial is $1-p$, can be approximated by a normal distribution.



You can determine the mean, μ, and standard deviation, σ, of the normal distribution from the binomial distribution using the formulas you explored in Activity 2.

$$\mu = n \times p \qquad \sigma = \sqrt{n \times p \times (1-p)}$$

Of course, it is only appropriate to approximate the binomial distribution by a normal distribution if there are a sufficient number of trials. The rule of thumb is if $n \times p \geq 5$ and $n \times (1-p) \geq 5$, it is appropriate to use a normal distribution. If this is not the case, you must solve the problem using procedures for binomial distributions only.

Next, you will apply the normal approximation in problem-solving settings.

Example

Use the normal approximation to estimate the probability of obtaining at least 8 heads when a fair coin is tossed 20 times.

Solution

When a fair coin is tossed, the probability of a head is $p = 0.5$.

Because the coin is tossed 20 times, $n = 20$.

Determine $n \times p$ and $n \times (1 - p)$ to check if it is appropriate to use the normal approximation.

$$\begin{aligned} n \times p &= 20 \times 0.5 \\ &= 10 \end{aligned} \qquad \begin{aligned} n \times (1 - p) &= 20 \times (1 - 0.5) \\ &= 10 \end{aligned}$$

Since $n \times p \geq 5$ and $n \times (1 - p) \geq 5$, it is appropriate to use the normal approximation.

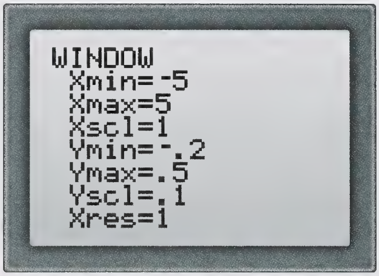
Next, determine the mean and standard deviation.

$$\begin{aligned} \mu &= n \times p \\ &= 20 \times 0.5 \\ &= 10 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{n \times p \times (1 - p)} \\ &= \sqrt{20 \times 0.5 \times (1 - 0.5)} \\ &= \sqrt{20 \times 0.5 \times 0.5} \\ &\doteq 2.236 \end{aligned}$$

Since you are interested in at least 8 heads, determine the *z*-score for $x = 8$.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &\doteq \frac{8 - 10}{2.236} \\ &\doteq \frac{-2}{2.236} \\ &\doteq -0.89 \end{aligned}$$

Use the following window settings to determine areas under the standard normal curve between *z*-scores.

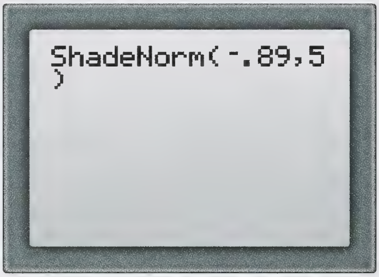


```
WINDOW
 Xmin=⁻5
 Xmax=5
 Xscl=1
 Ymin=⁻.2
 Ymax=.5
 Yscl=.1
 Xres=1
```

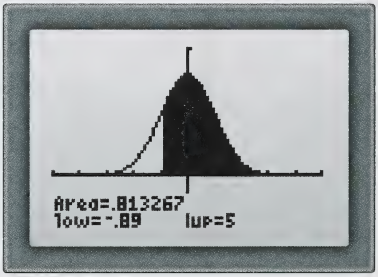
Use the Shade Normal feature of your graphing calculator to determine the area under the standard normal curve. **Remember:** Make sure any previous drawing is cleared off the calculator display. Press 2nd [DRAW] 1 (1:ClrDraw) ENTER.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() (–) • 8 9 , 5) ENTER



```
ShadeNorm(⁻.89,5
)
```



```
Area=.813267
low=⁻.89     up=5
```

$\therefore\ P(z > -0.89) \doteq 0.81$

An estimate of the probability of obtaining at least 8 heads when a fair coin is tossed 20 times is 0.81.

1. Turn to pages 126 and 127 of the textbook and complete exercises 1, 2, and 3 of "Investigation: Calculating Probabilities."

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 110–115.

Turn to pages 5 and 6 of Assignment Booklet 3B and answer questions 8 and 9.

Note: The Alberta Learning Diploma Examination uses the following to determine if it is appropriate to use a normal approximation to a binomial distribution. (It is different than the one used in the textbook.)

> If the following two conditions are met, you can use a normal approximation to a binomial distribution to solve problems.
>
> $n \times p \geq 5$ and $n \times (1-p) \geq 5$

This condition is less restrictive than the condition used in the textbook. This allows you to use the normal approximation more often.

Example

A multiple-choice test consists of 25 questions, each of which has 4 choices. To pass the test, a student must answer at least 13 questions correctly. What is the probability of passing the test by guessing only?

Solution

The probability of guessing the correct answer on any question is $p = 0.25$.

Because the test has 25 questions, $n = 25$.

Determine $n \times p$ and $n \times (1-p)$ to check if it is appropriate to use the normal approximation.

$$\begin{aligned} n \times p &= 25 \times 0.25 \\ &= 6.25 \end{aligned} \qquad \begin{aligned} n \times (1-p) &= 25 \times (1-0.25) \\ &= 25 \times (0.75) \\ &= 18.75 \end{aligned}$$

Since $n \times p \geq 5$ and $n \times (1-p) \geq 5$, it is appropriate to use the normal approximation.

Next, determine the mean and standard deviation.

$$\begin{aligned}\mu &= n \times p \\ &= 25 \times 0.25 \\ &= 6.25\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{25 \times 0.25 \times (1-0.25)} \\ &= \sqrt{25 \times 0.25 \times 0.75} \\ &\doteq 2.165\end{aligned}$$

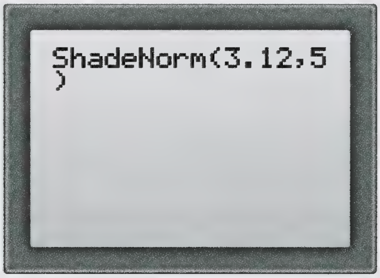
Determine the z-score for $x = 13$.

$$\begin{aligned}z &= \frac{x-\mu}{\sigma} \\ &\doteq \frac{13-6.25}{2.165} \\ &\doteq 3.12\end{aligned}$$

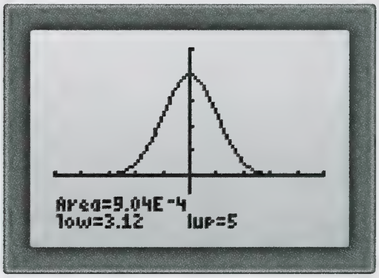
Using the same window settings as in the preceding example, use the Shade Normal feature of your graphing calculator to determine the probability of answering at least 13 questions correctly.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() 3 • 1 2 , 5) ENTER



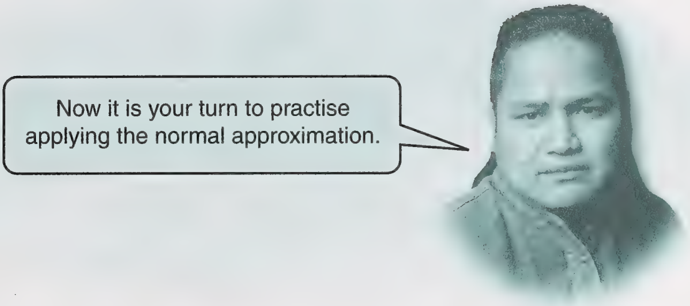
```
ShadeNorm(3.12,5
)
```



Area=9.04E-4
low=3.12 up=5

$\therefore P(z \geq 3.12) \doteq 0.0009$

The probability of passing the test by guessing alone is approximately 0.0009.


Now it is your turn to practise applying the normal approximation.

2. **a.** Suggest some reasons why a normal approximation to the binomial distribution should not be used if $n \times p$ or $n \times (1-p)$ is less than 5.

   b. Describe some factors that affect how close an estimate of the probability is to the actual probability.

3. Turn to pages 130 and 131 of the textbook and answer exercises 4, 5, and 6 of "Exercises: Checking Your Skills." **Note:** In exercise 4, the question should read "at most 600 ones," not 600 heads.

Compare your responses with the suggested answers in the Appendix, Activity 5, pages 115–118.

Turn to pages 7 and 8 of Assignment Booklet 3B and answer question 10.

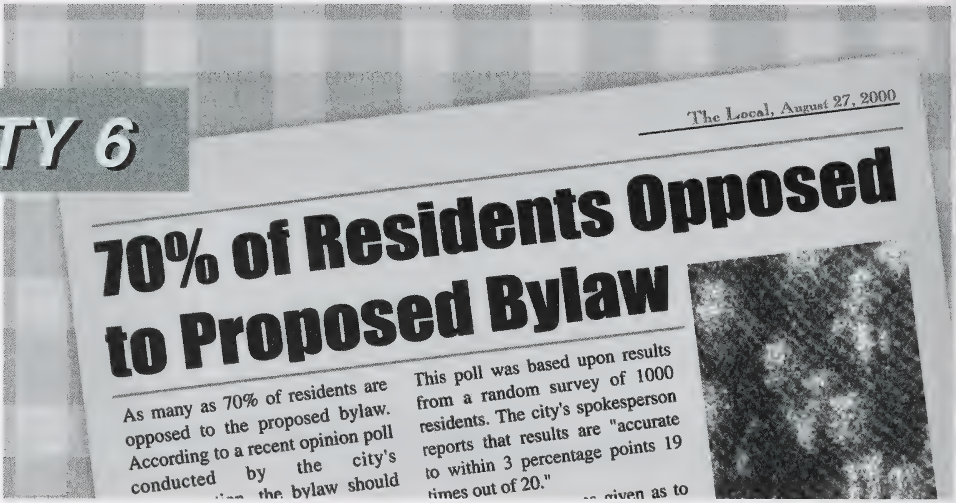
Looking Back

In this activity, you explored the normal approximation of the binomial distribution.

4. Turn to page 131 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 5, page 119.

ACTIVITY 6



The Local, August 27, 2000

70% of Residents Opposed to Proposed Bylaw

As many as 70% of residents are opposed to the proposed bylaw. According to a recent opinion poll conducted by the city's the bylaw should

This poll was based upon results from a random survey of 1000 residents. The city's spokesperson reports that results are "accurate to within 3 percentage points 19 times out of 20."

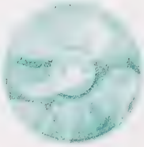
given as to

Confidence Intervals

A headline in a local newspaper reports that "70% of Residents Opposed to Proposed Bylaw." The accompanying article states that the headline was based on an opinion poll the city administration conducted. The article also states that the poll was based on a random survey of 1000 residents and that the results are considered to be "accurate to within three percentage points nineteen times out of twenty."

How can the newspaper be so confident about the accuracy of this survey? How do they know that 95% $\left(\frac{19}{20}\right)$ of the time this or similar surveys would be accurate within $\pm 3\%$?

In this activity, you will investigate **confidence intervals** for data that can be approximated or modelled by a normal distribution. Like the reported results of the opinion poll, a confidence interval is a range of values within which the mean of the data would fall within a stated percentage of the time.



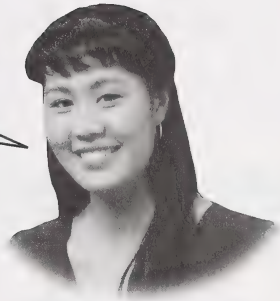
For more information, view the segment *Opinion Polls* on the Applied Mathematics 30 CD. This segment introduces the application of the normal distribution to solve problems involving confidence intervals for large samples arising from opinion polls.

Turn to page 134 of the textbook and read the introductory paragraphs of Tutorial 3.6, "Confidence Intervals."

1. Complete exercises 1, 2, and 3 of "Investigation: Find the Symmetric Interval Under the Standard Normal Curve That Contains 95% of the Data" on page 135 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 119–121.

Consider the following example involving an opinion poll and confidence intervals.



Example

Based on a survey of 250 decided voters, a polling firm reports that 64% are in favour of the bylaw and 36% are opposed to the bylaw.

a. For the 250 decided voters the polling firm surveyed, calculate the mean and the standard deviation of "yes" voters? Interpret these values.

b. Construct a 95% confidence interval for the percentage of voters who are in favour of the bylaw.

Solution

a. Calculate the mean and the standard deviation.

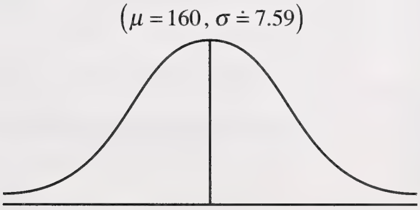
$$n = 250 \text{ and } p = 64\% = 0.64$$

$$\therefore \mu = n \times p = 250 \times 0.64 = 160$$

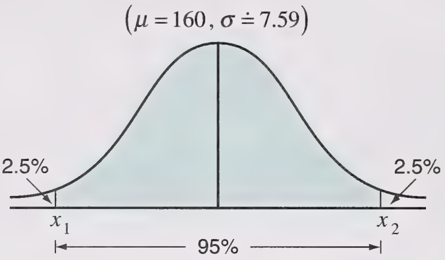
$$\sigma = \sqrt{n \times p \times (1-p)} = \sqrt{250 \times 0.64 \times (1-0.64)} = \sqrt{250 \times 0.64 \times 0.36} \doteq 7.59$$

The mean is 160 and the standard deviation is approximately 7.59.

If the polling firm repeated the survey by asking another 250 decided voters about the bylaw, it would expect 160 of the 250 people polled to say they supported the bylaw. However, they would not be surprised to find 162, 157, or some other number close to 160 that support the bylaw. As a matter of fact, if they repeated the survey over and over again, they would expect to find the number of supporters would be normally distributed with a mean of 160 and a standard deviation of 7.59.



b. You must determine the range of numbers of yes respondents that includes 95% of this distribution.



As you discovered in the Investigation on page 135 of the textbook, the 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma \\ &\doteq 160 - 1.96(7.59) \\ &\doteq 145\end{aligned}$$

The lower bound of the interval is 145. Only 2.5% of the distribution is less than this value.

$$\begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma \\ &\doteq 160 + 1.96(7.59) \\ &\doteq 175\end{aligned}$$

The upper bound of the interval is 175. Only 2.5% of the distribution is greater than this value.

Therefore, 95% of the time, or 19 times out of 20, the number of "yeses" in a survey of 250 voters would lie between approximately 145 and 175. As percentages, these numbers of "yeses" are

$$\frac{145}{250} \times 100 = 58\% \qquad \frac{175}{250} \times 100 = 70\%$$

Therefore, the 95% confidence interval for the percentage of "yes" votes is 58% to 70%, or $64\% - 6\%$ to $64\% + 6\%$.

The polling firm could report the results of its survey as follows:

> "Sixty-four percent of decided voters support the bylaw. These results are considered accurate within six percent, nineteen times out of twenty."

Turn to page 135 of the textbook and study the information following the Investigation. Then work through "Example 1: Construct a Confidence Interval" on page 136.

2. Answer exercise 1 of "Discussing the Ideas" on page 138 of the textbook.

Compare your responses with the suggested answers in the Appendix, Activity 6, page 121.

In the first example in this activity, you discovered that the confidence interval could be expressed as a proportion or a percentage on either side of the mean. Recall that the answer to this example stated, "Sixty-four percent of decided voters support the bylaw. These results are considered accurate within six percent, nineteen times out of twenty." The percentage $\pm 6\%$ is called the **margin of error**. Remember, to calculate the confidence interval in this example, you added and subtracted 1.96σ from the mean. When you divide the variation from the mean, $\pm 1.96\sigma$, by the sample size, n, you obtain the percent margin of error.

Turn to pages 136 and 137 of the textbook and read the information following Example 1 through to the beginning of Example 2.

The width of a 95% confidence interval grows if the number of samples grows. Remember that the 95% confidence interval contains all values within 1.96σ of the mean and that $\sigma = \sqrt{n \times p \times (1-p)}$. As the number of samples, n, increases, the expression under the square root will grow, thus making the standard deviation, σ, increase. This is shown in the following spreadsheet, where the probability of success is 0.625.

	A	B	C	D	E	F	G
1	Number of Samples	Number of Successes	Standard Deviation	95% Confidence Interval for Successes		Margin of Error (±)	Margin of Error as a Percentage (±)
2				Lower	Upper		
3	10	6	1.54919334	2.9636	9.0364	3.0364	30.3642%
4	50	31	3.43220046	24.2729	37.7271	6.7271	13.4542%
5	100	62	4.85386444	52.4864	71.5136	9.5136	9.5136%
6	500	312	10.83106643	290.7711	333.2289	21.2289	4.2458%
7	1000	625	15.30931089	594.9938	655.0062	30.0062	3.0006%
8	5000	3125	34.23265984	3057.9040	3192.0960	67.0960	1.3419%
9	10000	6250	48.41229183	6155.1119	6344.8881	94.8881	0.9489%

If you look at column F, Margin of Error $(\pm)$, you will see that the margin of error increases as the number of samples increases. At the same time, the percent margin of error decreases.

When working with margins of error, it is unusual to report the percent margin of error. In the exercises for this module, give your answers to questions regarding margin of error as the percent margin of error.

You can calculate the percent margin of error using either of these formulas:

- percent margin of error $= \pm\dfrac{1.96\sigma}{n} \times 100\%$
- percent margin of error $= \pm 1.96\sqrt{\dfrac{p \times (1-p)}{n}} \times 100\%$

Now, turn to pages 137 and 138 of the textbook and work through "Example 2: Construct the Confidence Interval and Calculate the Margin of Error."

3. Answer the following on pages 138 to 141 of the textbook.

  a. exercises 2 and 3 of "Discussing the Ideas"
  b. exercises 1.a., 1.b., 2, 5, and 7 of "Exercises: Checking Your Skills"
  c. exercise 10 of "Exercises: Extending Your Thinking"

Compare your responses with the suggested answers in the Appendix, Activity 6, pages 121–125.

Turn to pages 9 to 12 of Assignment Booklet 3B and answer question 11.

Looking Back

In this activity, you analysed survey results. You constructed confidence intervals and calculated margins of error.

4. Turn to page 141 of the textbook and answer "Communicating the Ideas."

Compare your response with the suggested answer in the Appendix, Activity 6, page 125.

Module Review

This module dealt with Chapter 3: Statistics in the *Addison-Wesley Applied Mathematics 12 Source Book.*

Turn to page 144 of the textbook and review the skills and concepts that were developed in this module. Also, read the important results and formulas you discovered.

Set	Data
A	26, 22, 14, 16, 21, 15, 19
B	36, 32, 24, 26, 31, 25, 29
C	52, 44, 28, 32, 42, 30, 38
D	5, 8, 12, 17, 23, 30, 38

1. Use the sets of data given to answer exercises 1.a. to 1.g.

a. What is the mean and standard deviation of the data in set A?

b. What is the mean and standard deviation of the data in set B?

c. What is the mean and standard deviation of the data in set C?

d. How does the data in set B differ from the data in set A? How do the mean and standard deviation of these two data sets compare?

e. How does the data in set C differ from the data in set A? How do the mean and standard deviation of these two data sets compare?

f. How will the mean and standard deviation of a set of data change if each data item is increased by a fixed amount, k? How will the mean and standard deviation of a set of data change if each data item is multiplied by a fixed factor, f?

g. i. What is the mean and standard deviation of the data in set D?

ii. If each data item in set D were tripled to become a new data set, named E, what data would be in set E? What would you predict the mean and standard deviation of the data in set E to be?

iii. If each data item in set D were increased by 5 to become a new data set, named F, what data would be in set F? What would you predict the mean and standard deviation of the data in set F to be?

iv. What mean and standard deviation does your calculator give for the data in sets E and F?

2. Turn to page 146 of the textbook and answer exercises 5, 6, and 8 of Part B of "What Should I Be Able to Do?"

Compare your responses with the suggested answers in the Appendix, Module Review, pages 126–130.

Turn to pages 13 to 21 of Assignment Booklet 3B and complete the Module Review Assignment.

If you had difficulties understanding the skills and concepts in Module 3: Statistics, it is recommended that you contact your teacher for some extra help activities. If you have a clear understanding of the skills and concepts in Statistics, you may wish to do the following enrichment activity. You may decide to do both.

Enrichment

In Activity 4, you solved problems involving normal distributions using *z*-scores and the Shade Normal feature on your graphing calculator. There is an alternate approach. You can use the Normal Cumulative Distribution feature (normalcdf) instead. This feature allows you to solve the same problems without having to convert to *z*-scores first; you simply enter the information as given in the problem.

Example

The results on a national mathematics competition were normally distributed with a mean of 70 and a standard deviation of 10. Determine the proportion of students who received marks

a. greater than 75 **b.** less than 60 **c.** between 60 and 75

Solution

a. Method 1: Using the Shade Normal Feature

First, convert $x = 75$ to a *z*-score.

$x = 75,\ \mu = 70,$ and $\sigma = 10$

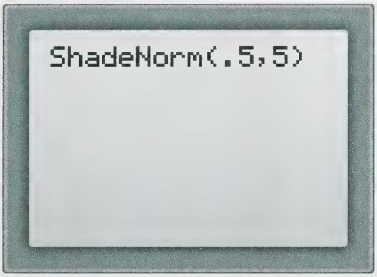
$$\begin{aligned}\therefore\ z &= \frac{x-\mu}{\sigma}\\ &= \frac{75-70}{10}\\ &= 0.5\end{aligned}$$

Because you are finding the proportion of students who received marks greater than 77, $z = 0.5$ is the lower bound. Choose $z = 5$ as an arbitrary upper bound.

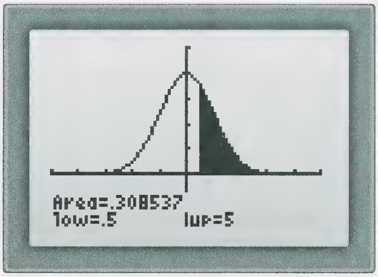
Now, press the following to obtain the percentage of students with marks greater than 70.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() . 5 , 5 ENTER



ShadeNorm(.5,5)



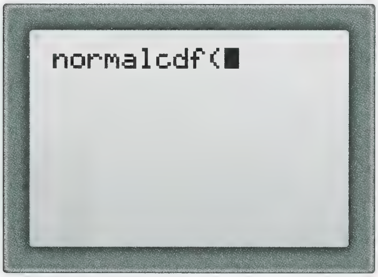
Area=.308537
low=.5 up=5

Approximately 30.85% of the students received marks greater than 75.

Method 2: Using the Normal Cumulative Distribution Feature

First, select the Normal Cumulative Distribution feature.

2nd [DISTR] 2 (2:normalcdf()



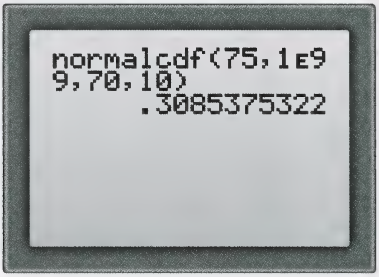
normalcdf(

This feature prompts you to enter the following information:

normalcdf (*lower bound, upper bound, mean, standard deviation*)

Here, the lower bound is 75. For the upper bound, you must choose an arbitrary large number to ensure you don't lose any of the distribution on the right. Use 1×10^{99}; it is entered in exponential form as 1 E 99. The mean is 70, and the standard deviation is 10.

7 5 , 1 2nd [EE] 9 9 , 7 0 , 1 0
) ENTER



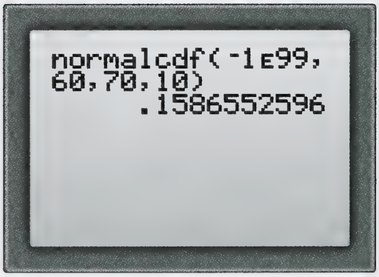
```
normalcdf(75,1E9
9,70,10)
          .3085375322
```

Approximately 30.85% of the students received marks greater than 75.

b. In this question you want the proportion to be less than 60. Therefore, 60 is the upper bound; and, again, you must choose a sufficiently small number for the lower bound, like -1×10^{99}.

Use the Normal Cumulative Distribution feature.

2nd [DISTR] 2 (2:normalcdf() (–) 1 2nd [EE] 9 9
, 6 0 , 7 0 , 1 0) ENTER

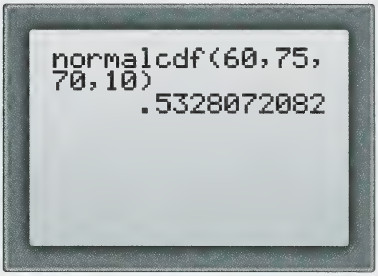


```
normalcdf(-1E99,
60,70,10)
          .1586552596
```

Approximately 15.87% of the students received marks less than 60.

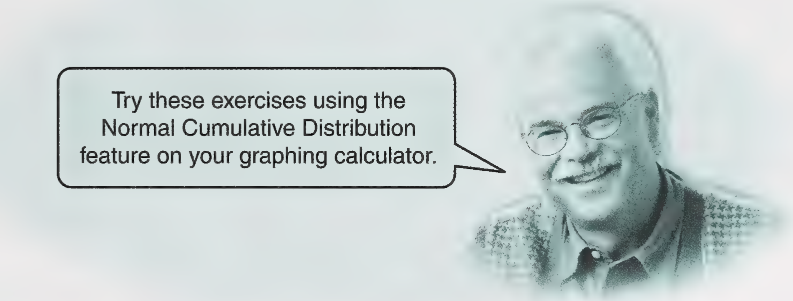
c. In this question, the lower bound is 60 and the upper bound is 75.

2nd [DISTR] 2 (2:normalcdf()
6 0 , 7 5 , 7 0
, 1 0) ENTER



```
normalcdf(60,75,
70,10)
      .5328072082
```

Approximately 53.28% of the students received marks between 60 and 75.


Try these exercises using the Normal Cumulative Distribution feature on your graphing calculator.

A manufacturer of light bulbs has determined the life of its product is normally distributed with a mean of 1000 h and a standard deviation of 100 h. Determine the probability that a given bulb will last

1. longer than 1175 h

2. less than 950 h

3. between 850 h and 1150 h

Compare your responses with the suggested answers in the Appendix, Module Review: Enrichment, pages 131–132.

MODULE PROJECT

Opinion Polls

Completing the Project

By now, you should have completed your initial research for the Module 3 project, Opinion Polls. By this time, you should have some idea of what direction your opinion poll will take.

Select the issue that interests you, and formulate a question that forces the respondents that you survey to answer *yes* or *no*. Then turn to page 132 of the textbook, read the information given, and answer questions 1 to 5 of "Conducting a Survey."

Keep a copy of your survey and the responses to these questions in the project section of your mathematics binder. You will use this information when you complete the project.

1. Answer exercise 10 of Part C of "What Should I Be Able to Do?" on page 147 of the textbook. In your review, comment on the correctness and completeness of the responses.

Compare your response with the suggested answer in the Appendix, Module Project, page 133.

Turn to page 142 and read "Reporting the Results." Then answer exercises 1 and 2. Keep a copy of your responses to these exercises in the project section of your mathematics binder. You will use this information when you complete the project.

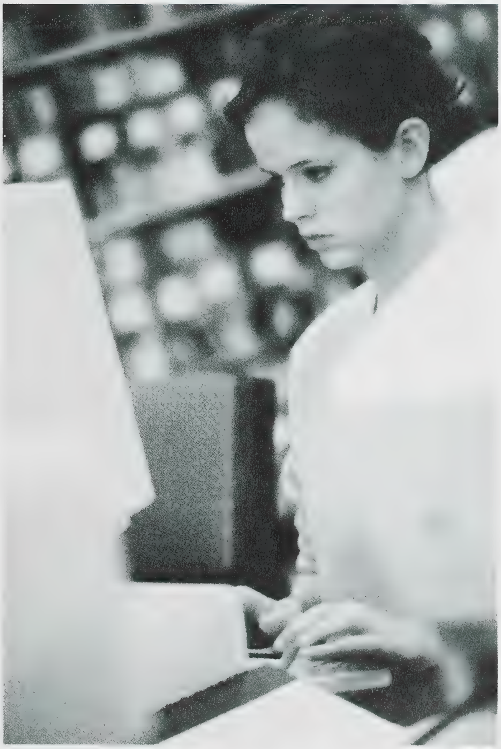
2. Answer exercise 11 of Part C of "What Should I Be Able to Do?" on page 148 of the textbook.

Compare your response with the suggested answer in the Appendix, Module Project, pages 133–134.

Module Project

Now, it's time to complete the Module 3 project, Opinion Polls. You may use your responses from the textbook exercises on pages 96, 132, 142, 147, and 148 to help you complete the project. (Your responses to these exercises should be in the project section of your mathematics binder.)

Turn to pages 22 to 28 of Assignment Booklet 3B and complete the module project.

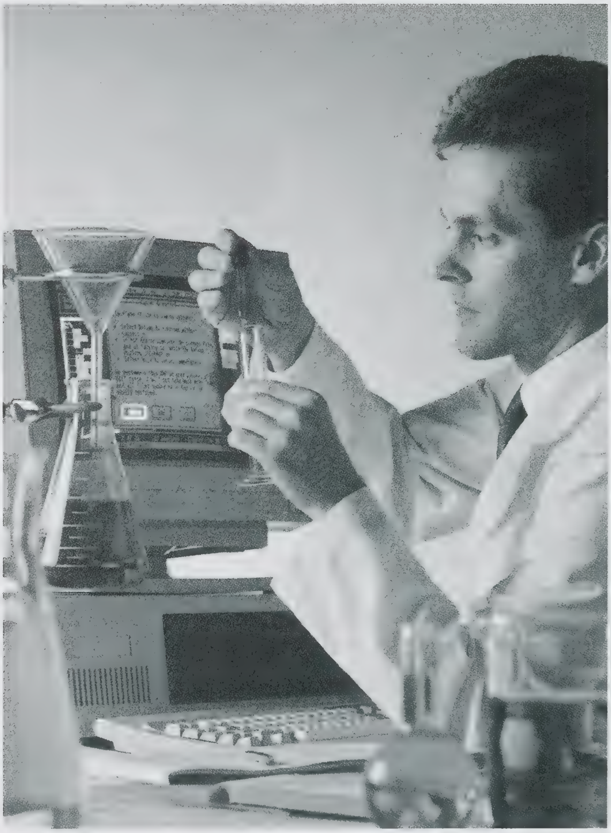


MODULE SUMMARY

In this module, you explored binomial and normal distributions and used these distributions to solve problems. You investigated the relationships among the means, standard deviations, and the proportions of data within these distributions. Using z-scores, you transformed normal distributions into standard normal distributions; and with the help of technology or tables, you calculated areas, proportions, and probabilities.

You then used the normal distribution to approximate the binomial distribution when sample sizes were large. In particular, you used the normal approximation to determine confidence intervals for survey results.

The normal distribution and the normal approximation have a variety of applications. Businesses, for example, use these statistical devices to determine warranty periods for their products. Polling firms use this branch of mathematics to determine the accuracy of their surveys. Medical, physical, and psychological researchers rely on the standard distribution (or bell curve) to interpret the results of their research or diagnostic testing.



APPENDIX

Glossary
Calculator Functions
Suggested Answers
Image Credits

Glossary

binomial distribution: the probability distribution for a binomial experiment

binomial experiment: an experiment with a fixed number of trials where each trial has the same probability of success (or failure)

confidence interval: the interval within which the value of a random variable is estimated to lie with a stated degree of probability

margin of error: the proportion added to and subtracted from the result to construct the confidence interval

mean: the average of the items in a data set

median: the middle value in a data set

mode: the value(s) in a data set that appears most often

normal curve: a bell-shaped, symmetrical curve representing the probability distribution for a random variable

It is also known as the Gaussian curve.

$$y=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}}$$

normal distribution: a frequency distribution that can be represented by the normal curve

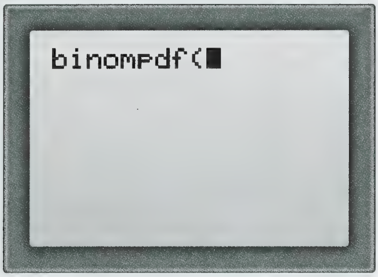
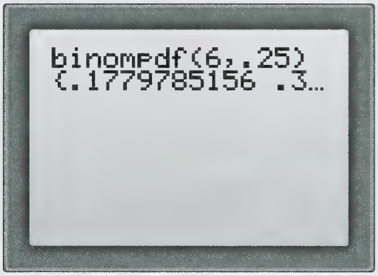
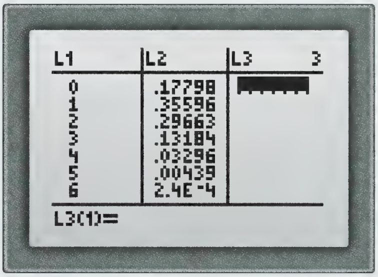
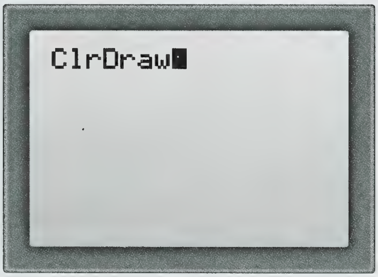
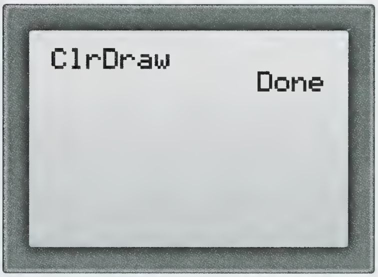
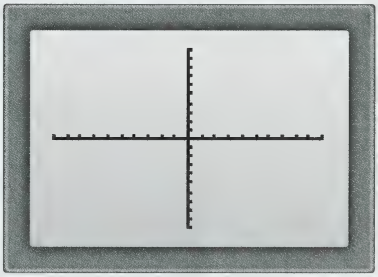
range: the difference between the largest and smallest values of a variable in the sample under consideration

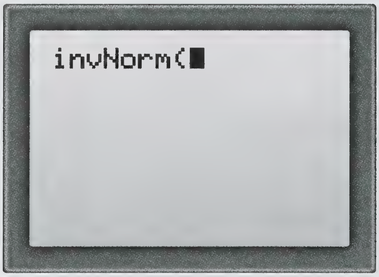
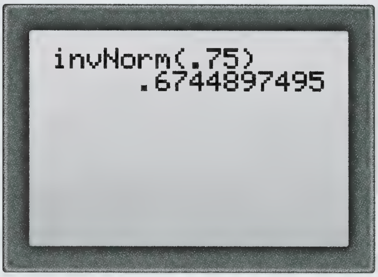
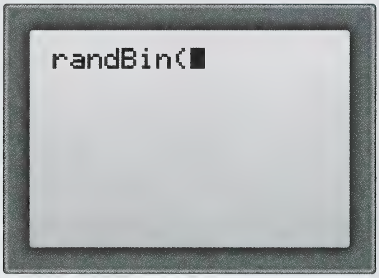
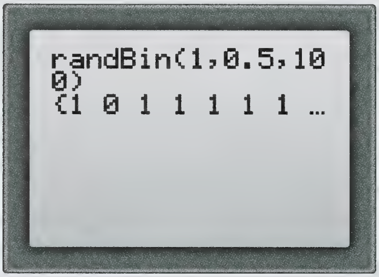
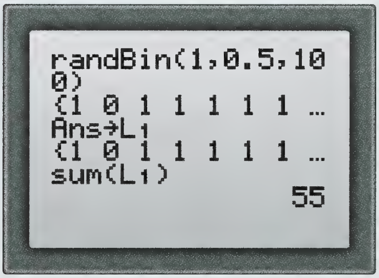
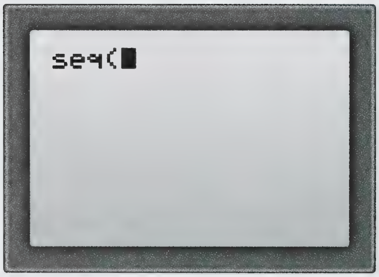
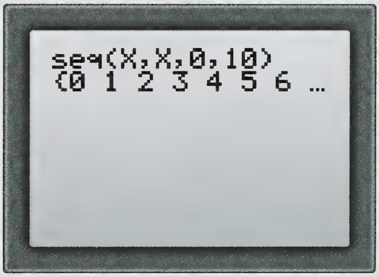
standard deviation: a measure of the dispersion of a frequency distribution

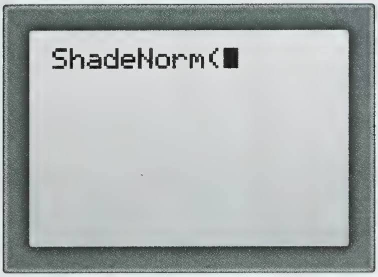
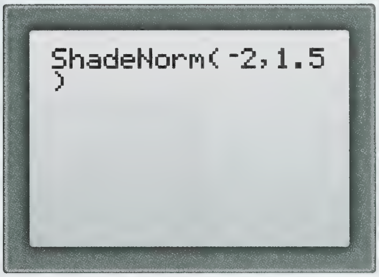
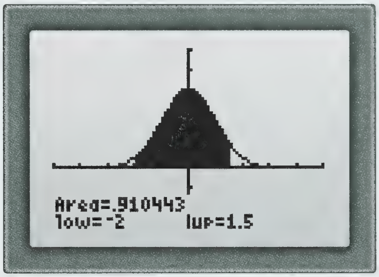
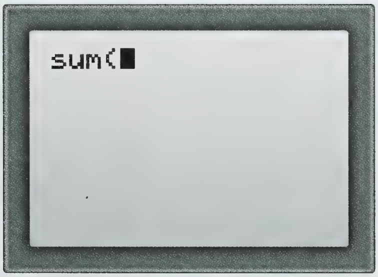
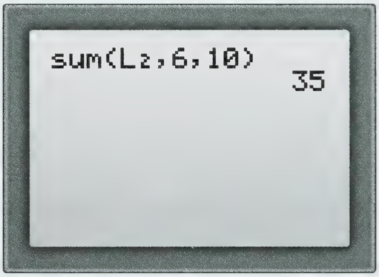
standard normal distribution: a normal distribution with a mean of 0 and a standard deviation of 1

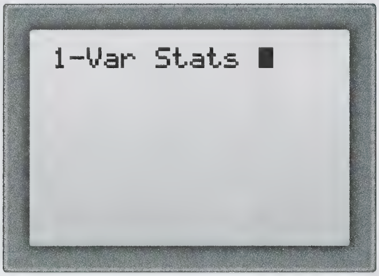
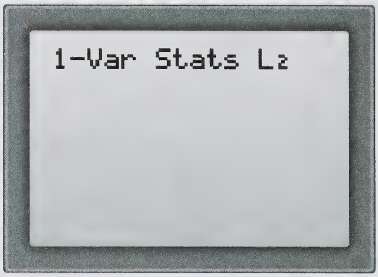
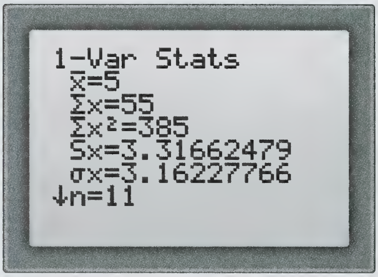
z-score: the independent variable in a standard normal distribution

Calculator Functions

Function	Keystrokes	Example
binompdf	2nd [DISTR] 0  binompdf(	 binompdf(6,.25) {.1779785156 .3…  L1 L2 L3 3 0 .17798 1 .35596 2 .29663 3 .13184 4 .03296 5 .00439 6 2.4E-4 L3(1)=
ClrDraw	2nd [DRAW] 1  ClrDraw	 ClrDraw Done 

invNorm	2nd [DISTR] 3  invNorm(	 invNorm(.75) .6744897495
randBin	Select the PRB menu. MATH ◀ 7  randBin(	 randBin(1,0.5,10 0) {1 0 1 1 1 1 1 …  randBin(1,0.5,10 0) {1 0 1 1 1 1 1 … Ans→L_1 {1 0 1 1 1 1 1 … sum(L_1) 55
seq	Select the OPS menu. 2nd [LIST] ▶ 5  seq(	 seq(X,X,0,10) {0 1 2 3 4 5 6 …

ShadeNorm	Select the DRAW menu. 2nd [DISTR] ▶ 1  ShadeNorm(	 ShadeNorm(⁻2,1.5)  Area=.910443 low=⁻2 up=1.5
sum	Select the MATH menu. 2nd [LIST] ◀ 5  sum(	 sum(L₂,6,10) 35

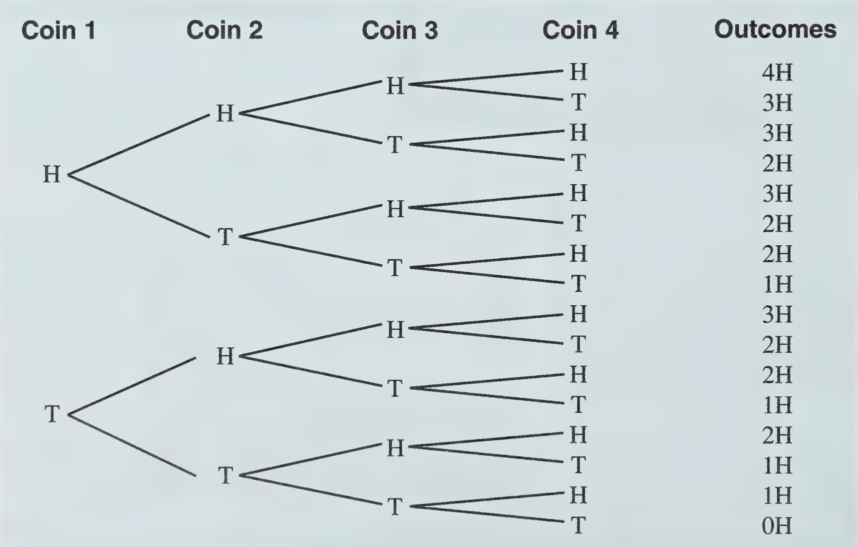
1–Var Stats	STAT ▶ 1  1-Var Stats ■	 1-Var Stats L2  1-Var Stats $\bar{x}=5$ $\Sigma x=55$ $\Sigma x^2=385$ Sx=3.31662479 σx=3.16227766 ↓n=11

Suggested Answers

Activity 1: Distributions

1. Textbook exercises 1 to 6 of “Investigation: Constructing Probability Distributions,” pp. 98 and 99

1.



2.

Outcome	Probability
4 Heads	$\frac{1}{16}$
3 Heads	$\frac{4}{16}$
2 Heads	$\frac{6}{16}$
1 Head	$\frac{4}{16}$
0 Heads	$\frac{1}{16}$

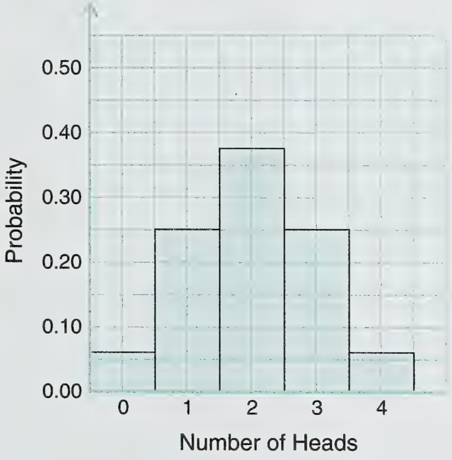
3. The lowest probability in the distribution from exercise 2 is $\frac{1}{16}$. The highest probability is $\frac{6}{16}$. Both of these values fall between the integers 0 and 1.

4.
$$\begin{aligned}\frac{1}{16}+\frac{4}{16}+\frac{6}{16}+\frac{4}{16}+\frac{1}{16}&=\frac{16}{16}\\&=1\end{aligned}$$

The sum of the probabilities in exercise 2 is 1.

5.

Probability Distribution of Heads
When 4 Coins Are Tossed



The histogram is very tall in the middle and decreases as the values move away from 2 heads.

6. Answers will vary. A sample experiment of flipping 4 coins 50 times is given.

Outcome	Tally	Frequency
4 Heads	|||	3
3 Heads	𝍸 |||	8
2 Heads	𝍸 𝍸 𝍸 𝍸 |	21
1 Head	𝍸 𝍸 |||	13
0 Heads	𝍸	5

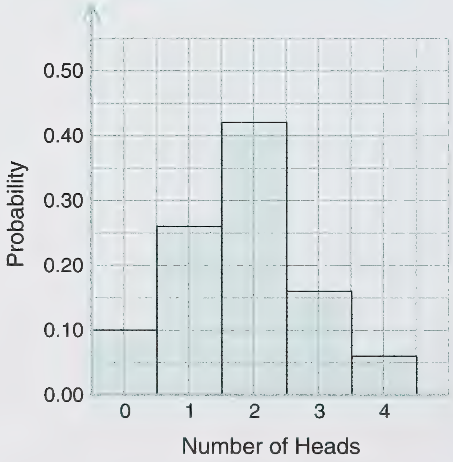
2. Textbook exercises 8 to 12 of "Investigation: Constructing Probability Distributions," p. 99

8. The lowest probability is $\frac{3}{50}$, and the highest probability is $\frac{21}{50}$. Both of these values fall between the integers 0 and 1.

$$\begin{aligned}\frac{3}{50}+\frac{8}{50}+\frac{21}{50}+\frac{13}{50}+\frac{5}{50}&=\frac{50}{50}\\&=1\end{aligned}$$

The sum of the probabilities is 1.

9. Experimental Probability of Heads When 4 Coins Are Tossed



10. Answers will vary. Sample data from four other groups are given.

Group 1

Outcome	Tally	Frequency
4 Heads	////	4
3 Heads	𝍸 𝍸 ///	13
2 Heads	𝍸 𝍸 𝍸 ///	18
1 Head	𝍸 𝍸 ///	13
0 Heads	//	2

Group 2

Outcome	Tally	Frequency
4 Heads	////	4
3 Heads	𝍸 𝍸 ////	14
2 Heads	𝍸 𝍸 𝍸 ////	19
1 Head	𝍸 //	7
0 Heads	𝍸 /	6

Group 3

Outcome	Tally	Frequency
4 Heads		0
3 Heads	𝍸 𝍸 𝍸	15
2 Heads	𝍸 𝍸 𝍸 𝍸 𝍸 //	27
1 Head	𝍸 //	7
0 Heads	/	1

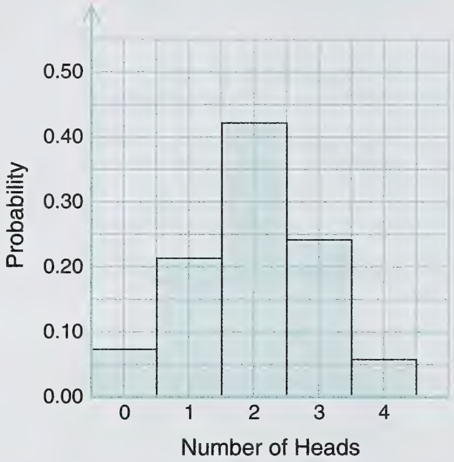
Group 4

Outcome	Tally	Frequency
4 Heads	///	3
3 Heads	𝍸 𝍸	10
2 Heads	𝍸 𝍸 𝍸 𝍸	20
1 Head	𝍸 𝍸 ///	13
0 Heads	////	4

The total experimental probability of each outcome for the five experiments is as follows.

Outcome	Experimental Probability
4 Heads	$\frac{14}{250}$
3 Heads	$\frac{60}{250}$
2 Heads	$\frac{105}{250}$
1 Head	$\frac{53}{250}$
0 Heads	$\frac{18}{250}$

Experimental Probability of Heads When 4 Coins Are Tossed



The experimental probabilities are approaching the probabilities as calculated in exercise 2.

11. The smallest probability is $\frac{14}{250}$, and the largest probability is $\frac{105}{250}$. Both of these values are between the integers 0 and 1.

$$\begin{aligned}\frac{14}{250}+\frac{60}{250}+\frac{105}{250}+\frac{53}{250}+\frac{18}{250}&=\frac{250}{250}\\&=1\end{aligned}$$

The sum of the probabilities is 1.

12. The estimated probability distribution becomes closer to the theoretical probability distribution as the number of experiments increases.

3. a. **Textbook exercises 1, 2, and 3 of "Discussing the Ideas," p. 102**

   1. The probability of an outcome is the quotient of the number of ways the outcome can occur and the total number of ways all outcomes can occur.

   $$\text{Probability} = \frac{\text{Number of ways the outcome can occur}}{\text{Total number of ways all outcomes can occur}}$$

   The least number of ways an outcome can occur is never, or 0 times; so, the probability must be 0 or greater. The largest number of ways an outcome can occur is always, or the total number of ways of obtaining all outcomes; so, the probability must be 1 or less. Therefore, the probability of any outcome for an event must be between 0 and 1, inclusive.

   2. The sum of the probabilities in a probability distribution is always 1 because all possible outcomes are included in the distribution. One of these outcomes must occur; so, the sum of the probabilities is 1.

   3. You would estimate the probabilities instead of using theoretical values when you are unable to obtain the theoretical probabilities. Suppose you needed the probability of a Canadian getting the flu next year. You could use data from previous years; but since the flu virus is constantly mutating, it would only be an estimate. The theoretical value cannot be known if the precise conditions are not known. Polling at election time is another example of using estimated probabilities to predict future events.

b. **Textbook exercises 2, 4, 6.a., and 6.b. of "Exercises: Checking Your Skills," pp. 103 and 104**

   2. a. Distribution 2 shows a uniform distribution. The probability of selecting any data at random is $\frac{1}{7}$.

      b. Distributions 1 and 4 are binomial distributions. Each distribution illustrates a fixed number of independent trials in which the outcomes are classified as successes or failures. In Distribution 1, there are 5 trials (children), where the success on each trial is the birth of a girl. In Distribution 4, there are 20 trials, where the success on each trial is a student under the age of 17.

      c. Distribution 3 is neither a uniform distribution (probabilities are not equal) nor a binomial distribution (several different events can occur, thus are not independent binomial trials).

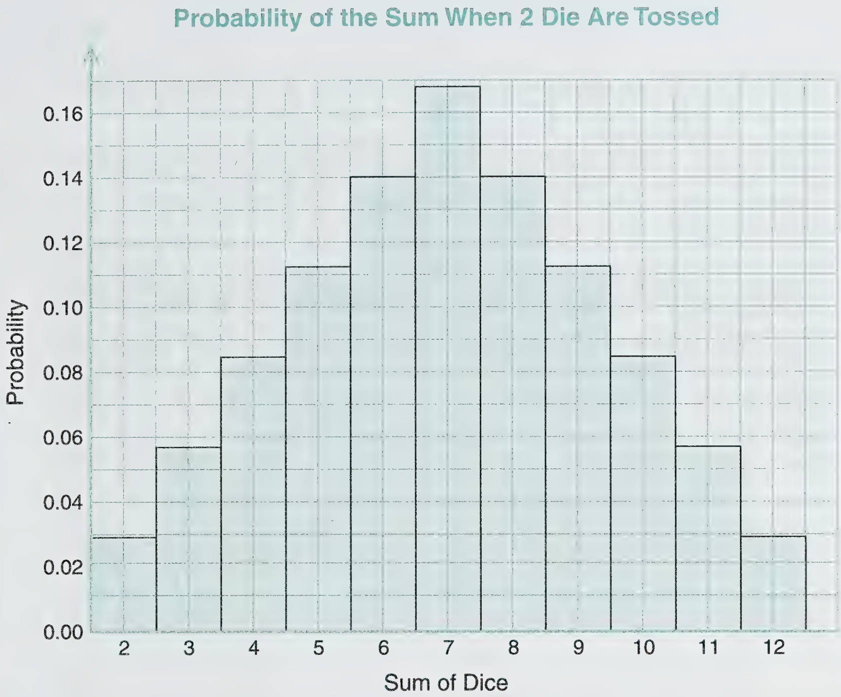
4. **a.** The sample space for the sum of two rolled dice is shown in the following chart.

	1	[illegible]	3	[illegible]	5	[illegible]
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. The experiment is not a binomial experiment. On any toss of a pair of dice there are eleven possibilities for the sum, not only two—success and failure—as occurs in a binomial experiment.

c.

Outcome	[illegible]	Probability
2	1	$\frac{1}{36} = 0.02\overline{7}$
3	2	$\frac{2}{36} = 0.0\overline{5}$
4	3	$\frac{3}{36} = 0.08\overline{3}$
5	4	$\frac{4}{36} = 0.\overline{1}$
6	5	$\frac{5}{36} = 0.13\overline{8}$
7	6	$\frac{6}{36} = 0.1\overline{6}$
8	5	$\frac{5}{36} = 0.13\overline{8}$
9	4	$\frac{4}{36} = 0.\overline{1}$
10	3	$\frac{3}{36} = 0.08\overline{3}$
11	2	$\frac{2}{36} = 0.0\overline{5}$
12	1	$\frac{1}{36} = 0.02\overline{7}$
Total	36	1


Probability of the Sum When 2 Die Are Tossed

6. **a.** There are three items in the sample space. They are shown in the following table.

Outcome	Probability
Green	$\frac{75}{140}$
Amber	$\frac{5}{140}$
Red	$\frac{60}{140}$

b. This is not a binomial experiment because there are three outcomes.

4. Textbook exercise "Communicating the Ideas" p. 104

A uniform distribution is a probability distribution in which the probabilities of several different events are equal. The histogram of this distribution is rectangular.

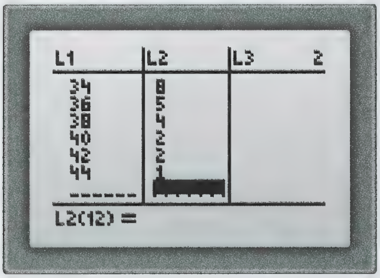
A binomial distribution is the probability distribution of a binomial experiment. A binomial experiment consists of a number of independent trials in which the outcomes are classified as successes or failures. This distribution has a bell-shaped histogram.

There are also probability distributions that are neither uniform nor binomial. Their histograms are not rectangular and may or may not be bell-shaped.

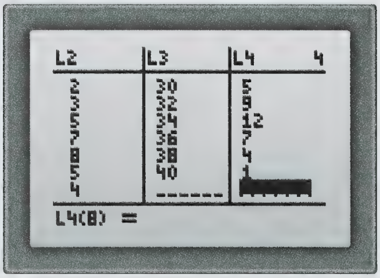
Activity 2: Mean and Standard Deviation

1. Textbook exercises 1 to 4 of “Investigation 1: Compare Sets of Data,” pp. 105 and 106

1. Press STAT 1 (1:Edit…) and enter the Value data from Data Set 1 in list L1 and the corresponding Frequency data in list L2. Then enter the Value data from Data Set 2 in list L3 and the corresponding Frequency data in list L4.

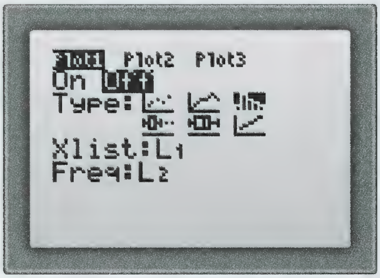


```
L1      L2      L3    2
  34     8
  36     5
  38     4
  40     2
  42     2
  44     1
 ------
L2(12) =
```

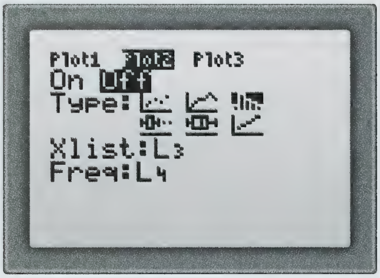


```
L2      L3      L4    4
  2      30     5
  3      32     9
  5      34     12
  7      36     7
  8      38     4
  5      40     1
  4     ------
L4(8)  =
```

Press 2nd [STAT PLOT] and set up Plot 1 and Plot 2. Select the histogram as the graph type in each case. Do not turn them on at this point.

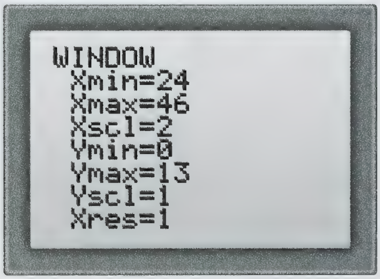


```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L1
Freq:L2
```



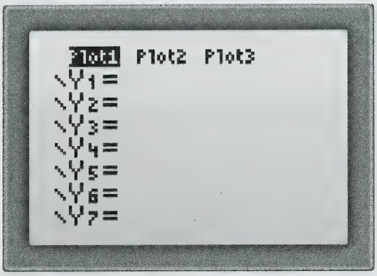
```
Plot1 Plot2 Plot3
On Off
Type:
Xlist:L3
Freq:L4
```

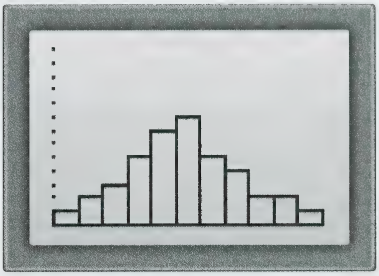
Use the following window settings.



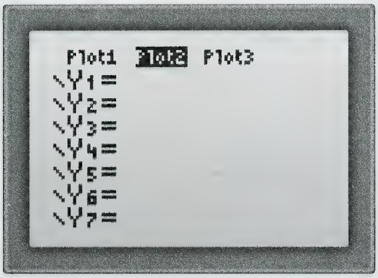
```
WINDOW
 Xmin=24
 Xmax=46
 Xscl=2
 Ymin=0
 Ymax=13
 Yscl=1
 Xres=1
```

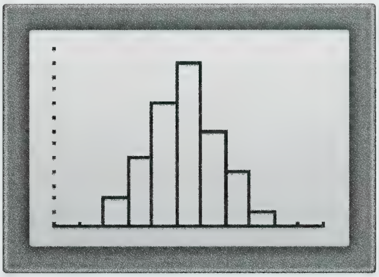
Now, press (Y=) and select "Plot 1" to graph the histogram for Data Set 1. (Place the cursor over "Plot 1" and press (ENTER).)





Graph the histogram for Data Set 2. Press (Y=); deselect "Plot 1"; and select "Plot 2."



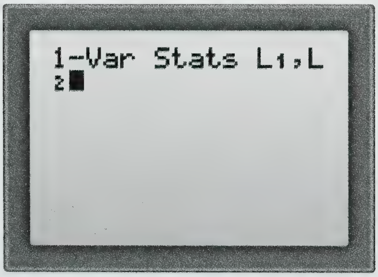


2. Both histograms are bell-shaped. The range of data values for Data Set 2 is narrower than that for Data Set 1. The frequencies for Data Set 2 are comparatively larger than those for Data Set 1.

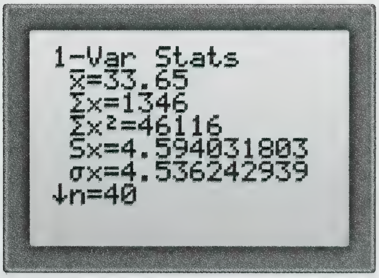
3. Calculate the mean and standard deviation for Data Set 1.

Select the CALC menu.

(STAT) (▶) (1) (1:1–Var Stats) (2nd) [L1] (,) (2nd) [L2] (ENTER)



```
1-Var Stats L1,L
2█
```



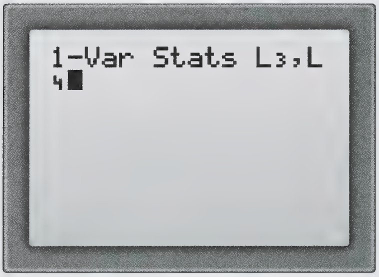
```
1-Var Stats
 x̄=33.65
 Σx=1346
 Σx²=46116
 Sx=4.594031803
 σx=4.536242939
↓n=40
```

For Data Set 1, the mean is 33.65 and the standard deviation is approximately 4.536.

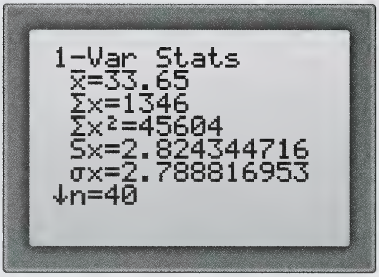
Calculate the mean and standard deviation for Data Set 2.

Select the CALC menu

STAT ▶ 1 2nd [L3] , 2nd [L4] ENTER



```
1-Var Stats L3,L
4
```



```
1-Var Stats
 x̄=33.65
 Σx=1346
 Σx²=45604
 Sx=2.824344716
 σx=2.788816953
↓n=40
```

For Data Set 2, the mean is 33.65 and the standard deviation is approximately 2.789.

4. The histogram for Data Set 1 is more spread out than the histogram for Data Set 2. Data Set 1 has the higher standard deviation. Therefore, the larger the standard deviation, the more spread out the data is.

2. **Textbook exercises 1, 3, and 4 of "Exercises: Checking Your Skills," pp. 108 and 109**

1. The data in 1.b. is likely to have the greatest standard deviation. Its data is the most spread out.

   The data in 1.c. is likely to have the least standard deviation. Its data is tightly clustered about the mean.

3. The data in 3.a. is likely to have the greatest standard deviation. It is a uniform distribution and the data are least clustered.

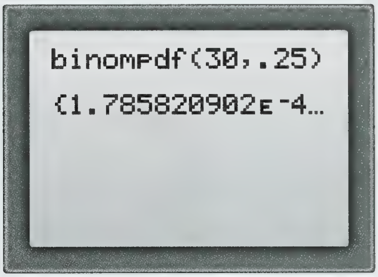
   The data in 3.c. is likely to have the least standard deviation. Its data is tightly clustered about the mean.

4. The 35 randomly selected people at the shopping mall would have the greater standard deviation for the number of successful free throws. The range of abilities would be greater for this group than it would be for the members of the high school basketball league.

3. Textbook exercises 1 to 5 of "Investigation 2: Mean and Standard Deviation of a Binomial Distribution," pp. 106 and 107

1. Construct the probability distribution of the 30-question, multiple-choice test.

2nd [DISTR] 0 (0:binompdf () 3 0 ,
. 2 5) ENTER

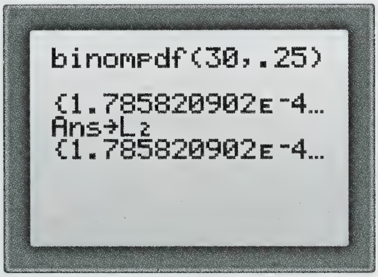


```
binompdf(30,.25)
{1.785820902E-4…
```

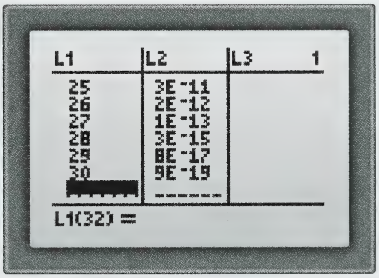
2. Store the results from exercise 1 in list L2 by pressing the following.

STO➔ 2nd [L2] ENTER

Then enter the values 0, 1, 2, …, 30 in list L1. (You may need to clear list L1 first.)



```
binompdf(30,.25)
{1.785820902E-4…
Ans→L2
{1.785820902E-4…
```

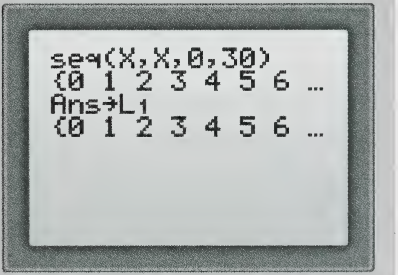


```
L1     L2      L3     1
25     3E-11
26     2E-12
27     1E-13
28     3E-15
29     8E-17
30     9E-19
       ------
L1(32) =
```

Note: You can easily fill list L1 with the values 0 to 30 by pressing the following.

Select the CALC menu.

2nd [LIST] ▶ 5 (5:seq ()
X,T,Θ,n , X,T,Θ,n , 0 ,
3 0) STO➔ 2nd [L1]
ENTER



```
seq(X,X,0,30)
{0 1 2 3 4 5 6 …
Ans→L1
{0 1 2 3 4 5 6 …
```

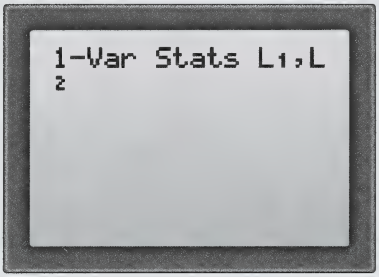
Press STAT 1 (1:Edit…) to make sure the values 0 to 30 appear in list L1.

Activity 2 (continued)

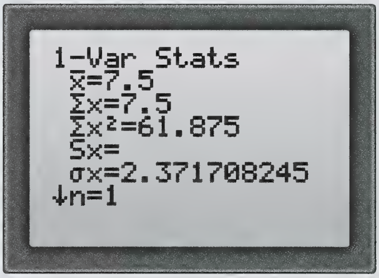
Finally, calculate the mean and standard deviation.

Select the CALC menu.

STAT ▶ 1 (1:1–Var Stats) 2nd [L1] , 2nd [L2] ENTER



```
1-Var Stats L₁,L
₂
```



```
1-Var Stats
 x̄=7.5
 Σx=7.5
 Σx²=61.875
 Sx=
 σx=2.371708245
↓n=1
```

The mean is 7.5, and the standard deviation is approximately 2.372.

3. The probability of success on any trial is 0.25, and there are 30 trials.

$\therefore\ 30 \times 0.25 = 7.5$

This matches the mean found in exercise 2.

4. The probability of failure on any trial is 0.75.

$$\begin{aligned}\therefore\ \sqrt{7.5 \times 0.75} &= \sqrt{5.625} \\ &\doteq 2.371\,708\,245\end{aligned}$$

This is the same as the standard deviation calculated in exercise 2.

5. The mean can be found by multiplying the number of trials by the probability of success on each trial. This can be expressed as follows, where n is the number of trials and p is the probability of success.

$$\text{Mean} = n \times p$$

The standard deviation can be found by taking the square root of the product of the number of trials times the probability of success times the probability of failure. This can be expressed as follows, where n is the number of trials, p is the probability of success, and $(1-p)$ is the probability of failure.

$$\text{Standard deviation} = \sqrt{n \times p \times (1-p)}$$

4. **a. Textbook exercises 1, 2, and 3 of “Discussing the Ideas,” p. 108**

   1. It is reasonable to use $n \times p$ to represent the mean of a binomial distribution because this product is the expected number of successes in n trials of a binomial experiment for which p is the probability of success on each trial. For example, on a 100-item, multiple-choice test with 4 choices for each item, you would expect to get $100 \times 0.25 = 25$ items correct by guessing alone.

   2. The small standard deviation suggests that the fasteners in crate B are of uniform size. A small standard deviation suggests that the distribution is tightly clustered about the mean. A small variation implies uniformity.

   3. Answers will vary. Sample answers are given.

      a. A good situation for data to have a high standard deviation is choosing players for a hockey team. If the players come with a wide range of skills, selecting players for the team will appear objective and less arbitrary than it would if all the candidates were equal in ability.

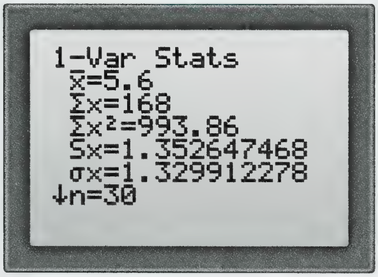
      b. A situation in which it would be good for data to have a low standard deviation involves sizes of parts manufactured for the auto industry. Uniformity of size is important so that purchased replacement parts will fit.

   b. Textbook exercises 5 and 8 of “Exercises: Checking Your Skills,” pp. 109 and 110

   5. The video game scores of 40 randomly selected people at a hockey game would likely have a greater standard deviation than the scores of 40 randomly selected people at an arcade. The skills of the players at the arcade would be more uniform than the skills of those at the hockey game. Some of the people at the hockey game may have never played a video game.

   8. **a.** Use your graphing calculator to determine the mean and standard deviation for each brand of battery.

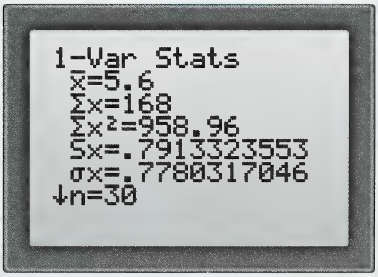
Brand X



```
1-Var Stats
 x̄=5.6
 Σx=168
 Σx²=993.86
 Sx=1.352647468
 σx=1.329912278
↓n=30
```

The mean is 5.6, and the standard deviation is about 1.33.

Brand Y



```
1-Var Stats
 x̄=5.6
 Σx=168
 Σx²=958.96
 Sx=.7913323553
 σx=.7780317046
↓n=30
```

The mean is 5.6, and the standard deviation is about 0.78.

b. Because the means are the same, the better battery is the one with the smaller standard deviation. With a smaller standard deviation, you can be more certain that if you purchase this battery, it will last close to 5.6 years.

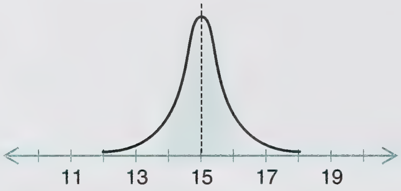
Activity 2 (continued)

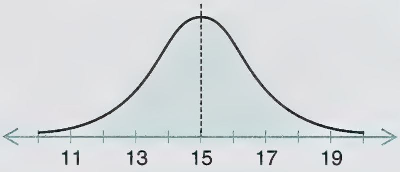
5. Textbook exercise "Communicating the Ideas," p. 110

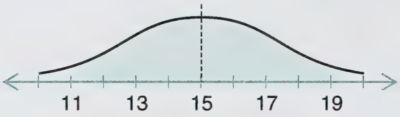
Answers will vary. A sample answer is given.

The mean is a single number that describes the average of a set of data. The standard deviation is a number that describes how spread out the data is.

The following graphs show three distributions with the same mean but different standard deviations. Each area corresponds to 2 standard deviations above and below the mean.

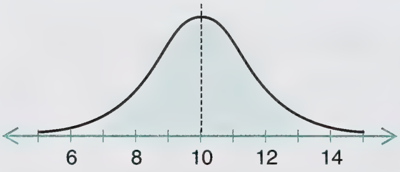


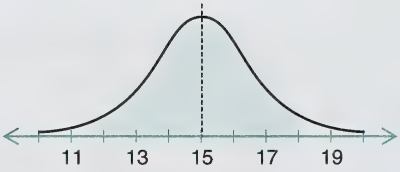


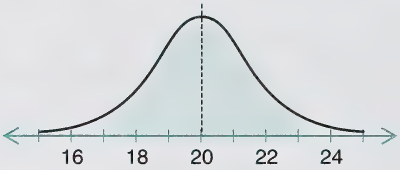


If you know that the mean length of life for a particular breed of cat is 15 years, this gives you some idea of how long the average cat of that breed will live. If you don't know the standard deviation, you will have no idea how close to that average most cats of that breed will live.

The following graphs show three distributions with the same standard deviation but different means. Each area corresponds to 2 standard deviations above and below the mean.







If you know that the standard deviation for the length of life is 1 year for this breed of cat but don't know the mean, you also don't know how long a particular cat is likely to live. It takes both the mean and the standard deviation to be able to make reasonable predictions about a set of data.

Activity 3: Normal Distributions

1. **a. Textbook exercises 2 to 5 of "Discussing the Ideas," p. 116**

2. a. If the mean remained the same and the standard deviation increased from 8 to 16, the normal curve would be wider and less tall.

b. If the mean increased from 25 to 75 but the standard deviation remained the same, the normal curve would have exactly the same shape but would move 50 units to the right.

c. If the mean increased to 50 and the standard deviation increased to 11, the curve would shift 25 units to the right and would be stretched horizontally. The curve would be wider and not as tall.

3. a. A faster way to estimate the range is to use the fact that about 99.7% of the data lies within 3 standard deviations of the mean.

$$\begin{aligned}\therefore \text{ Range} &\doteq (\mu+3\sigma)-(\mu-3\sigma)\\ &\doteq \mu+3\sigma-\mu+3\sigma\\ &\doteq 6\sigma\end{aligned}$$

Simply multiply the standard deviation by 6 to estimate the range of the distribution.

$$\begin{aligned}\therefore \text{ Range} &\doteq 6\sigma\\ &\doteq 6(0.76)\\ &\doteq 4.56\end{aligned}$$

b. The range cannot be determined exactly because the largest and smallest values in the data set are not given in the problem.

4. Since the sum of the probabilities of a probability distribution is 1, you can use the normal curve, which encloses an area of 1, to estimate probabilities.

5. Answers will vary. A sample answer is given.

Problems that involve the mean, standard deviation, percentage and proportion of data, and area under the normal curve can be solved with the information about a normal distribution that is presented in this Tutorial.

b. Textbook exercises 1 and 2.a. of "Exercises: Checking Your Skills," p. 116

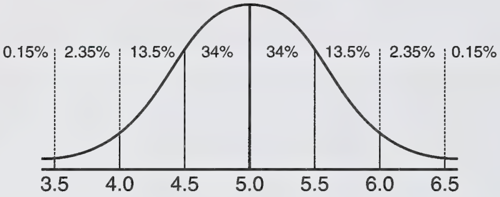
1. a. The masses of a bundle of 200 wooden fence posts is most likely represented by graph ii. The normal curve is narrow, suggesting a small standard deviation and, therefore, uniform posts in the bundle.

b. The ages of people in a park on a Sunday afternoon is most likely represented by graph iii. The normal curve is wide, suggesting a large standard deviation and, therefore, people of all ages.

c. The marks on a standardized achievement test are most likely represented by graph i. It appears as the typical normal curve.

2. a.

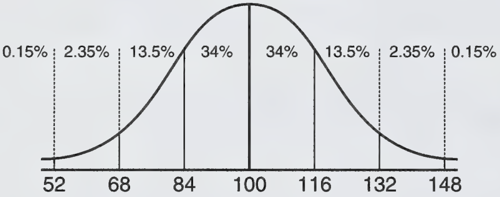
$(\mu = 5.0, \sigma = 0.5)$



2. Textbook exercises 3, 5, 6, 7, and 8 of "Exercises: Checking Your Skills," pp. 117 and 118

3. Draw a sketch to help visualize the problem.

$(\mu = 100, \sigma = 16)$



a. The probability that a randomly selected Canadian will have an IQ less than 100 is 0.50, or 50%.

b. $50\% + 34\% \doteq 84\%$

The probability that a randomly selected Canadian will have an IQ less than 116 is about 0.84, or 84%.

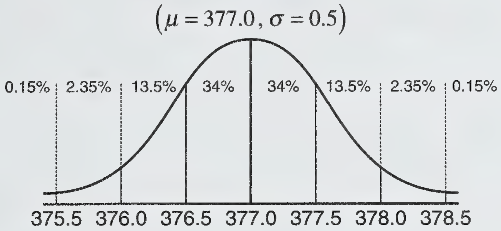
c. $100\% - 84\% = 16\%$

The probability that a selected Canadian will have an IQ greater than 116 is about 0.16, or 16%.

d. $34\% + 34\% = 68\%$

The probability that a selected Canadian will have an IQ between 84 and 116 is about 0.68, or 68%.

5. Draw a sketch to help visualize the problem.



a. $0.15\% + 2.35\% \doteq 2.50\%$

Approximately 2.50% of the cans will have less than 376.0 mL.

b. Approximately 0.15% of the cans will have less then 375.5 mL.

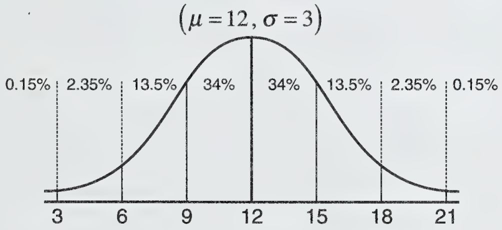
c.
$$\begin{aligned} 0.15\% \text{ of } 50\,000 &= 0.0015 \times 50\,000 \\ &= 75 \end{aligned}$$

About 75 cans will be rejected in a production run of 50 000.

d. $100\% - 2.5\% \doteq 97.5\%$

The probability that a randomly selected can contains a volume greater than 376.0 mL is about 0.975, or 97.5%.

6. Draw a sketch to help visualize the problem. **Note:** The mean is 12 months (1 year), and the standard deviation is 3 months.

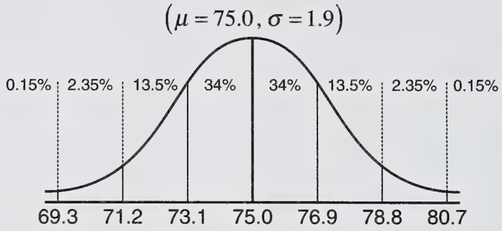


$34\% + 34\% + 13.5\% \doteq 81.5\%$

The probability that a randomly selected light bulb will have a life between 9 and 18 months is about 0.815, or 81.5%.

Activity 3 (continued)

7. Draw a sketch to help visualize the problem.



a. $2.35\% + 0.15\% = 2.50\%$

The probability that a ball selected at random will be "fast" (bounce above 78.8 cm) is about 0.025, or 2.5%.

b. The approximate range of heights will be from 3 standard deviations below the mean to 3 standard deviations above the mean.

$$\therefore \text{Range} \doteq 80.7 - 69.3 \qquad \text{or} \qquad \text{Range} \doteq 6\sigma$$
$$\doteq 11.4 \qquad\qquad\qquad \doteq 6(1.9)$$
$$\doteq 11.4$$

8. About 95% of the data is within 2 standard deviations of the mean. Because 95% of the sacks have masses between 56.4 kg and 65.6 kg,

$$\text{2 standard deviations from mean} = 61.0 - 56.4 \quad \longleftarrow \text{ or } 65.6 - 61.0$$
$$2\sigma = 4.6$$
$$\sigma = 2.3$$

The standard deviation is 2.3 kg.

3. Project Book exercises 1, 2, and 3.a. of "Getting Started," p. 48

1. To have all the options available, Up-To-Date Jeans Co. will have to create $2 \times 2 \times 2 \times 2 = 16$ styles in each size.

2. Logo-Patch Jeans will have $6 \times 5 \times 4$ or 120 stamp sets to advertise assuming the patches must be in order (hexagon, pentagon, square). If the patches can be in any order, the number of stamp sets increases by a factor of $3! = 6$ to $120 \times 6 = 720$. If not all patches have to be in a stamp set, the number of sets will increase yet again.

3. a. **Retail**

$$\text{Cost per pair of jeans} = 2.5 \text{ m} \times \$7.99/\text{m}$$
$$= \$19.98$$

Wholesaler Without Discount

Determine the number of medium-sized jeans that can be made out of an 80-m bolt of denim.

$$\begin{aligned}\text{Number of jeans} &= \frac{80\text{ m}}{2.5\text{ m}}\\ &= 32\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Cost per pair of jeans} &= \frac{\text{Cost of denim}}{\text{Number of jeans}}\\ &= \frac{\$400}{32}\\ &= \$12.50\end{aligned}$$

Wholesaler with Discount

The discount is given when more than 5 bolts of denim are purchased. Therefore, at least 6 bolts of denim must be purchased to get the discount.

First, determine the amount and cost of purchasing 6 bolts of denim.

$$\begin{aligned}\text{Amount of denim} &= 80\text{ m}\times 6\\ &= 480\text{ m}\end{aligned}$$

$$\begin{aligned}\text{Cost of denim} &= 85\%\ \text{of}\left(\$400\times 6\right)\\ &= 0.85\times \$2400\\ &= \$2040\end{aligned}$$

Now, determine the number of medium-sized jeans that can be made out of six 80-m bolts of denim.

$$\begin{aligned}\text{Number of jeans} &= \frac{480\text{ m}}{2.5\text{ m}}\\ &= 192\end{aligned}$$

$$\begin{aligned}\therefore\ \text{Cost per pair of jeans} &= \frac{\text{Cost of denim}}{\text{Number of jeans}}\\ &= \frac{\$2040}{192}\\ &= \$10.63\end{aligned}$$

The cost per pair of jeans is most expensive when buying the denim at retail prices; the cost is least expensive when six or more bolts of denim are purchased.

4. Textbook exercise "Communicating the Ideas," p. 118

A normal distribution is a bell-shaped distribution characteristic of distributions of height, mass, intelligence, and other physical or mental traits. Normal distributions can be used to determine proportions of the population between particular measures for each of these traits.

Activity 4: Standard Normal Distribution

1. **a. Textbook exercises 1 and 2 of "Discussing the Ideas," p. 123**

1. In the calculation of a z-score, it is important that you find $x-\mu$ and not $\mu-x$ so the proper sign (positive or negative) is assigned to the z-score. If x is greater than the mean, the z-score should be positive; if x is less than the mean, the z-score should be negative. If you find $\mu-x$, this will be reversed.

2. In the formula $z=\frac{x-\mu}{\sigma}$, the difference $x-\mu$ tells you how far the data value, x, is above or below the mean, μ. When the difference $x-\mu$ is divided by σ, you will obtain the number of standard deviations the data value, x, lies from the mean.

b. Textbook exercise 1 of "Exercises: Checking Your Skills," pp. 123 and 124

1. **a.** $x=2.5$, $\mu=2.4$, and $\sigma=0.1$

$$\begin{aligned}\therefore\ z&=\frac{x-\mu}{\sigma}\\&=\frac{2.5-2.4}{0.1}\\&=\frac{0.1}{0.1}\\&=1\end{aligned}$$

b. $x=85$, $\mu=78$, and $\sigma=12$

$$\begin{aligned}\therefore\ z&=\frac{x-\mu}{\sigma}\\&=\frac{85-78}{12}\\&=\frac{7}{12}\\&\doteq 0.58\end{aligned}$$

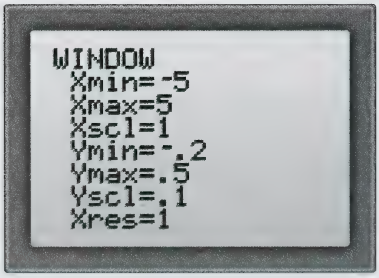
c. $x=1.8$, $\mu=2.7$, and $\sigma=0.3$

$$\begin{aligned}\therefore\ z&=\frac{x-\mu}{\sigma}\\&=\frac{1.8-2.7}{0.3}\\&=\frac{-0.9}{0.3}\\&=-3\end{aligned}$$

d. $x=6375$, $\mu=8500$, and $\sigma=250$

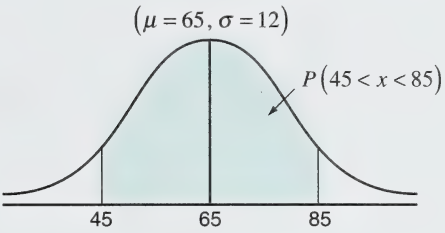
$$\begin{aligned}\therefore\ z&=\frac{x-\mu}{\sigma}\\&=\frac{6375-8500}{250}\\&=\frac{-2125}{250}\\&=-8.5\end{aligned}$$

2. Use the following window settings for each part of this question.



```
WINDOW
 Xmin=-5
 Xmax=5
 Xscl=1
 Ymin=-.2
 Ymax=.5
 Yscl=.1
 Xres=1
```

a. Sketch the distribution.



Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 45$ and $x = 85$ into z-scores.

For $x = 45$,

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ z_{45} &= \frac{45-65}{12} \\ &= \frac{-20}{12} \\ &\doteq -1.67 \end{aligned}$$

For $x = 85$,

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ z_{85} &= \frac{85-65}{12} \\ &= \frac{20}{12} \\ &\doteq 1.67 \end{aligned}$$

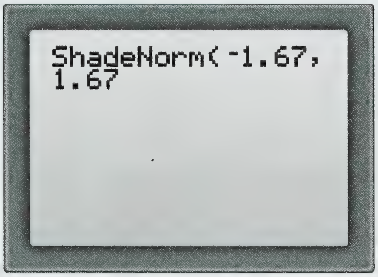
Method 1: Using a Graphing Calculator

2nd [DISTR] ▶ 1 (1:ShadeNorm() (–) 1 • 6 7 , 1 • 6 7) ENTER

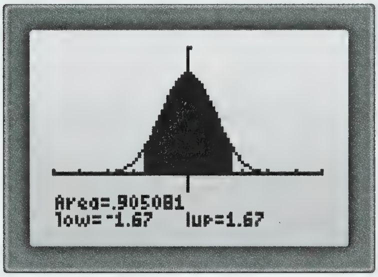
Select the DRAW menu.

lower bound

upper bound



ShadeNorm(⁻1.67,
1.67



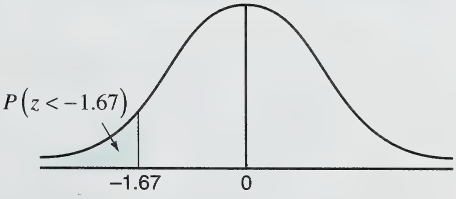
Area=.905081
low=⁻1.67 up=1.67

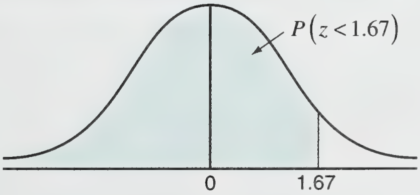
$\therefore P(45 < x < 85) \doteq 0.9051$

Method 2: Using Tables

For $z_{45} = -1.67$, $P(z < -1.67) = 0.0475$.

For $z_{85} = 1.67$, $P(z < 1.67) = 0.9525$.

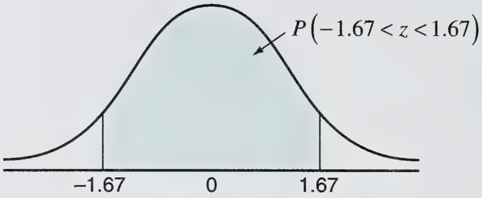




Activity 4 (continued)

Subtract to find the area between $z_{45} = -1.67$ and $z_{85} = 1.67$.

$$\begin{aligned} P(-1.67 < z < 1.67) &= P(z < 1.67) - P(z < -1.67) \\ &= 0.9525 - 0.0475 \\ &= 0.9050 \end{aligned}$$

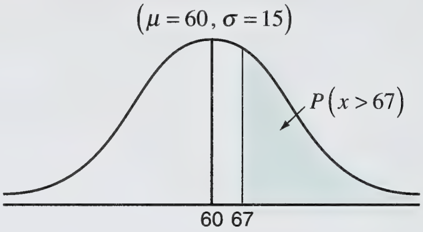


$\therefore\ P(45 < x < 85) \doteq 0.9050$

Note: The differences between the answers obtained using Method 1 and Method 2 are due to rounding. The values in the tables have been rounded to 4 decimal places.

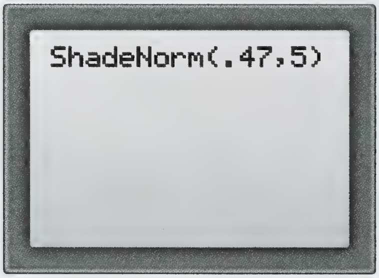
b. Sketch the distribution.

Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 67$ into a z-score.

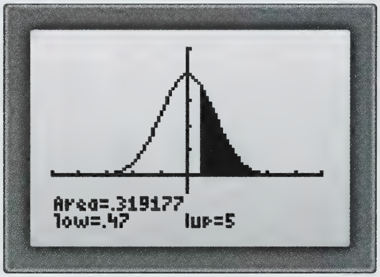


$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{67-60}{15} \\ &= \frac{7}{15} \\ &\doteq 0.47 \end{aligned}$$

Method 1: Using a Graphing Calculator



ShadeNorm(.47,5)



Area=.319177
low=.47 up=5

Remember: Use 5 as the upper bound in this case. Also, clear the preceding drawing on your calculator before proceeding. Press 2nd [DRAW] 1 (1:ClrDraw).

$\therefore\ P(x > 67) \doteq 0.3192$

Method 2: Using Tables

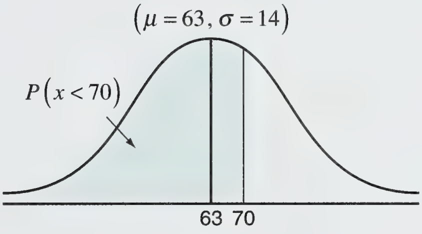
For $z = 0.47$, $P(z < 0.47) = 0.6808$.

$$\begin{aligned} \therefore\ P(z > 0.47) &= 1 - P(z < 0.47) \\ &= 1 - 0.6808 \\ &= 0.3192 \end{aligned}$$

$\therefore\ P(x > 67) \doteq 3192$

c. Sketch the distribution.

Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 70$ into a z-score.



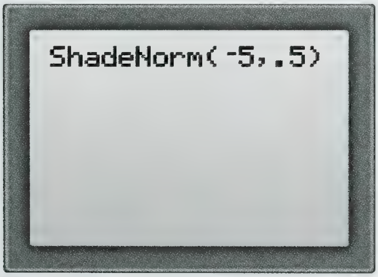
$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{70-63}{14} \\ &= \frac{7}{14} \\ &= 0.50 \end{aligned}$$

Method 1: Using a Graphing Calculator

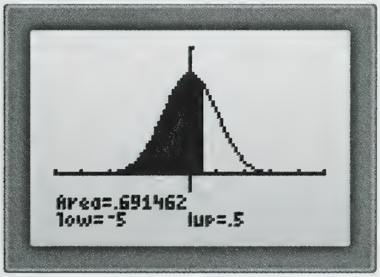
Using -5 as the lower bound, press the following.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() (–) 5 , • 5) ENTER



```
ShadeNorm(-5,.5)
```



```
Area=.691462
low=-5       up=.5
```

$\therefore\ P(x < 70) \doteq 0.6915$

Method 2: Using Tables

For $z_{70} = 0.5$, $P(z < 0.50) = 0.6915$.

$\therefore\ P(x < 70) \doteq 0.6915$

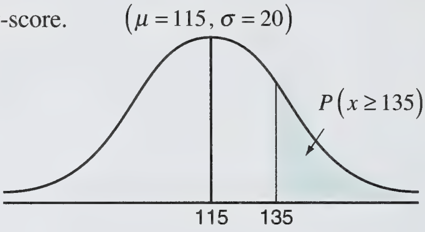
3. a. **Textbook exercise 3 of "Discussing the Ideas," p. 123**

3. You use -5 as the lower bound because almost all of the standard normal distribution lying to the left of the mean is included. The portion omitted is so small that the difference will not be enough to affect even the fourth decimal place of the area (or proportion). Also, -5 is convenient to display the distribution in the window settings.

b. **Textbook exercises 3, 5, 6.a., 7, and 9 of "Exercises: Checking Your Skills," pp. 124 and 125**

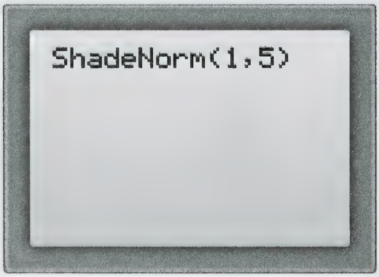
3. Sketch the distribution.

Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 135$ into a z-score.

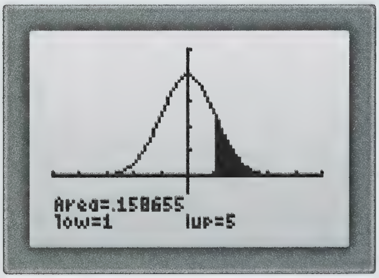


$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{135-115}{20} \\ &= \frac{20}{20} \\ &= 1 \end{aligned}$$

Method 1: Using a Graphing Calculator



```
ShadeNorm(1,5)
```



```
Area=.158655
low=1         up=5
```

$\therefore\ P(x \geq 135) \doteq 0.1587$

Method 2: Using Tables

For $z = 1.00$, $P(z < 1.00) = 0.8413$.

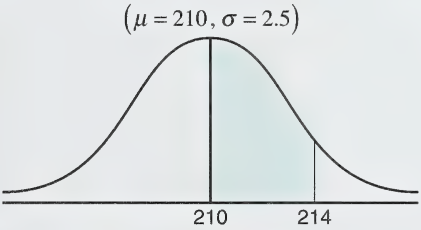
$$\begin{aligned} \therefore\ P(z \geq 1.00) &= 1 - P(z < 1.00) \\ &= 1 - 0.8413 \\ &= 0.1587 \end{aligned}$$

$$\therefore\ P(x \geq 135) \doteq 0.1587$$

The probability that a randomly selected student will receive an A is approximately 0.1587.

5. **a.** Sketch the distribution.

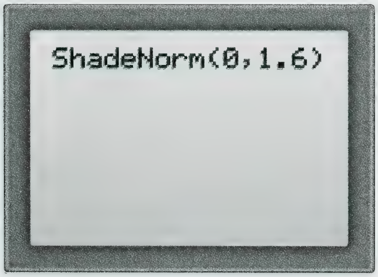
Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 214$ into a z-score.



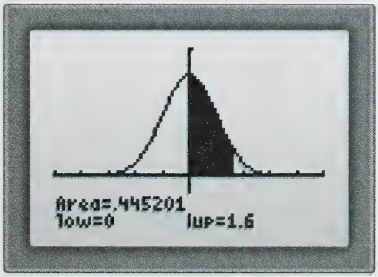
$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{214-210}{2.5} \\ &= \frac{4}{2.5} \\ &= 1.6 \end{aligned}$$

b. **Method 1: Using a Graphing Calculator**

Since 210 is the mean of the normal distribution, the lower bound should be entered as 0.



```
ShadeNorm(0,1.6)
```



```
Area=.445201
low=0        up=1.6
```

$$\therefore\ P(210 < x < 214) \doteq 0.4452$$

Method 2: Using Tables

For $z = 0$, $P(z < 0) = 0.5000$.

For $z = 1.6$, $P(z < 1.6) = 0.9452$.

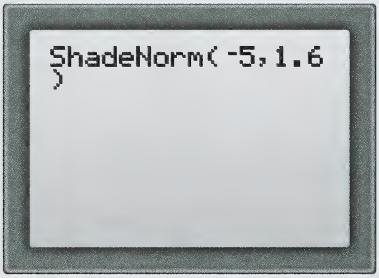
Subtract to find the area.

$$\begin{aligned} P(0 < x < 1.6) &= P(z < 1.6) - P(z < 0) \\ &= 0.9452 - 0.5000 \\ &= 0.4452 \end{aligned}$$

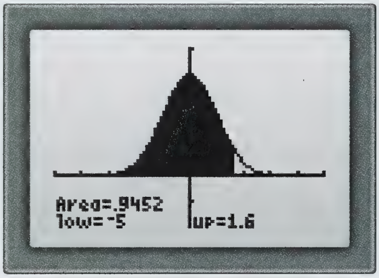
$\therefore\ P(210 < x < 214) \doteq 0.4452$

The probability that the mass of a randomly selected box will be between 210 g and 214 g is approximately 0.4552.

c. **Method 1: Using a Graphing Calculator**



```
ShadeNorm(-5,1.6
)
```



```
Area=.9452
low=-5         up=1.6
```

$\therefore\ P(x < 214) \doteq 0.9452$

Method 2: Using Tables

For $z = 1.6$, $P(z < 1.6) \doteq 0.9452$.

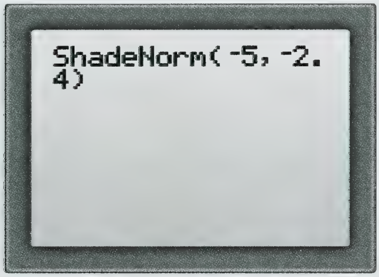
$\therefore\ P(x < 214) \doteq 0.9452$

The probability that the mass of a randomly selected box will be less than 214 g is approximately 0.9452.

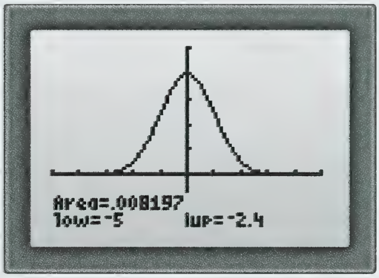
d. Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 204$ into a z-score.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{204-210}{2.5} \\ &= \frac{-6}{2.5} \\ &= -2.4 \end{aligned}$$

Method 1: Using a Graphing Calculator



```
ShadeNorm(-5,-2.
4)
```



```
Area=.008197
low=-5      up=-2.4
```

$\therefore\ P(x < 204) \doteq 0.0082$

Method 2: Using Tables

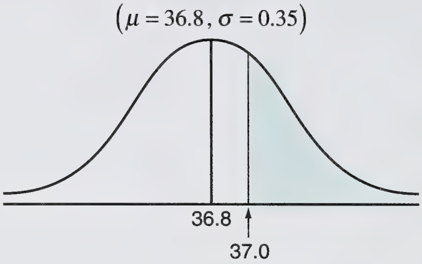
For $z = -2.4$, $P(z < -2.4) = 0.0082$.

$\therefore\ P(x < 204) \doteq 0.0082$

The probability that a box will be rejected is approximately 0.0082.

Activity 4 (continued)

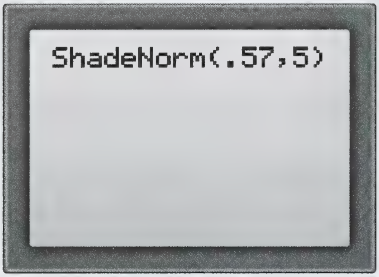
6. **a.** Sketch the distribution.



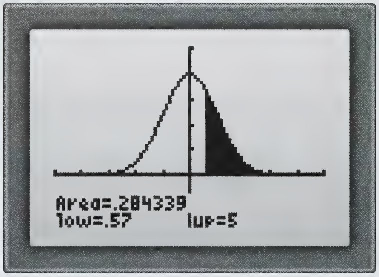
Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 37.0$ into a z-score.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{37.0-36.8}{0.35} \\ &= \frac{0.2}{0.35} \\ &\doteq 0.57 \end{aligned}$$

Method 1: Using a Graphing Calculator



ShadeNorm(.57,5)



Area=.284339
low=.57 up=5

$\therefore\ P(x > 37) \doteq 0.2843$

Method 2: Using Tables

For $z = 0.57$, $P(z < 0.57) = 0.7157$.

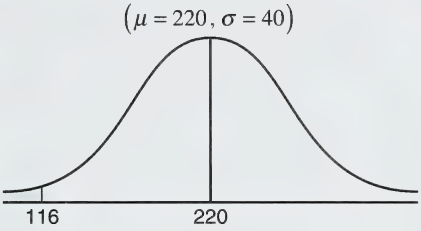
$$\begin{aligned} \therefore\ P(z > 0.57) &= 1 - P(z < 0.57) \\ &= 1 - 0.7157 \\ &= 0.2843 \end{aligned}$$

$\therefore\ P(x > 37) \doteq 0.2843$

Therefore, in a sample of 122 people, the expected number with temperatures about 37°C is

$$\begin{aligned} 122 \times P(x > 37) &\doteq 122(0.2843) \\ &\doteq 35 \end{aligned}$$

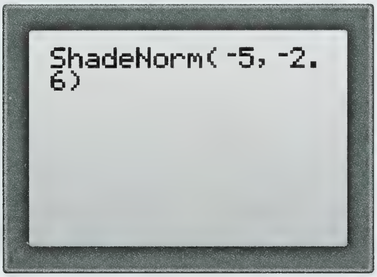
7. Sketch the distribution.



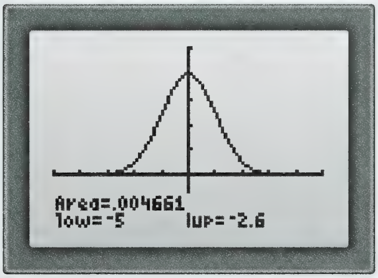
Use the formula $z = \frac{x-\mu}{\sigma}$ to convert a score of 116 into a z-score.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{116-220}{40} \\ &= \frac{-104}{40} \\ &= -2.6 \end{aligned}$$

Method 1: Using a Graphing Calculator



ShadeNorm(⁻5,⁻2.6)



Area=.004661
low=⁻5 up=⁻2.6

$\therefore\ P(x<116) \doteq 0.0047$

Method 2: Using Tables

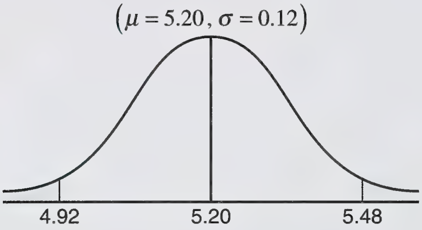
$P(z<-2.6) \doteq 0.0047$

$\therefore\ P(x<116) \doteq 0.0047$

The probability that a randomly selected student will be eligible for remediation is approximately 0.0047.

Activity 4 (continued)

9. Sketch the distribution.



Use the formula $z = \frac{x-\mu}{\sigma}$ to convert 4.92 and 5.48 into z-scores.

For $x = 4.92$,

$$z = \frac{x-\mu}{\sigma}$$

$$z_{4.92} = \frac{4.92 - 5.20}{0.12}$$

$$\doteq -2.33$$

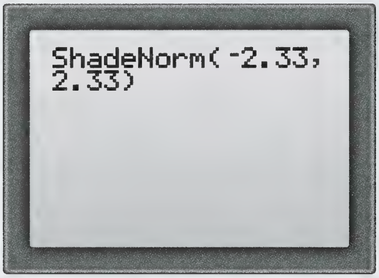
For $x = 5.48$,

$$z = \frac{x-\mu}{\sigma}$$

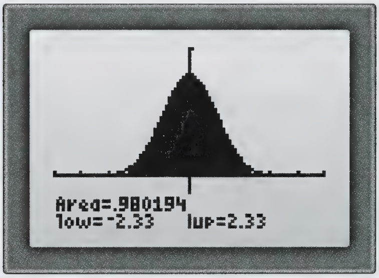
$$z_{5.48} = \frac{5.48 - 5.20}{0.12}$$

$$\doteq 2.33$$

Method 1: Using a Graphing Calculator



```
ShadeNorm(⁻2.33,
2.33)
```



```
Area=.980194
low=⁻2.33    up=2.33
```

$\therefore\ P(-2.33 < z < 2.33) \doteq 0.9802$

$\therefore\ P(z < -2.33 \text{ or } z > 2.33) \doteq 1 - 0.9802$

$\doteq 0.0198$

$\therefore\ P(x < 4.92 \text{ or } x > 5.48) \doteq 0.0198$

Method 2: Using Tables

For $z_{4.92} = -2.33$, $P(z < -2.33) \doteq 0.0099$.

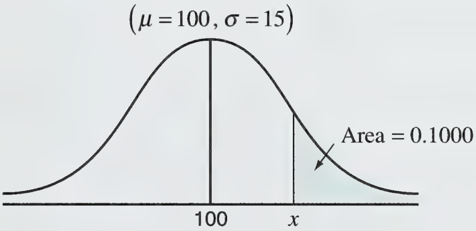
Because the distribution is symmetric, for $z_{5.48} = 2.33$, $P(z > 2.33) \doteq 0.0099$.

$$\therefore\ P(z < -2.33 \text{ or } z > 2.33) \doteq 0.0099 + 0.0099$$
$$\doteq 0.0198$$

$$\therefore\ P(x < 4.92 \text{ or } x > 5.48) \doteq 0.0198$$

The percent of the eggs that must be stored in another container is approximately 1.98%.

4. Sketch the distribution. Let x be the score above which 10% of the distribution lies. Therefore, x is the score below which 90% of the distribution lies.



Find the z-score corresponding to an area of 0.9000.

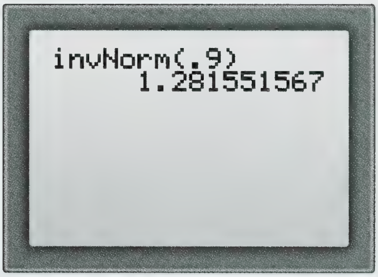
Method 1: Using a Graphing Calculator

Use the Inverse Normal Distribution feature on your graphing calculator.

2nd [DISTR] 3 (3:invNorm()

• 9) ENTER

$\therefore\ z \doteq 1.28$



```
invNorm(.9)
          1.281551567
```

Method 2: Using Tables

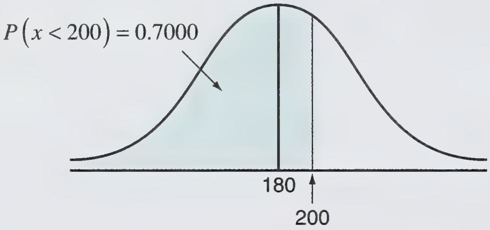
Look for an area closest to 0.9000 on the z-score tables. The closest value in the body of the tables is 0.8997, which corresponds to a z-score of 1.28.

Next, use $z=\frac{x-\mu}{\sigma}$ to find x.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ 1.28 &\doteq \frac{x-100}{15} \\ 1.28(15) &\doteq x-100 \\ 19.2 &\doteq x-100 \\ 100+19.2 &\doteq x \\ x &\doteq 119.2 \end{aligned}$$

An IQ greater than 119 is necessary to be in the top 10% of the population.

5. Sketch the distribution.



Find the z-score corresponding to an area of 0.7000.

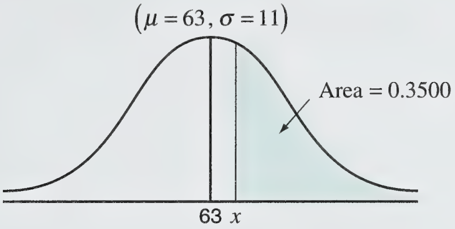
Look for an area closest to 0.7000. The closest value in the body of the tables is 0.6985, which corresponds to a z-score of 0.52.

Next, use $z=\frac{x-\mu}{\sigma}$ to find σ.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ 0.52 &\doteq \frac{200-180}{\sigma} \\ 0.52\sigma &\doteq 20 \\ \sigma &\doteq \frac{20}{0.52} \\ &\doteq 38.5 \end{aligned}$$

The standard deviation is approximately 38.5.

6. Sketch the distribution. Let x be Djon's mark.

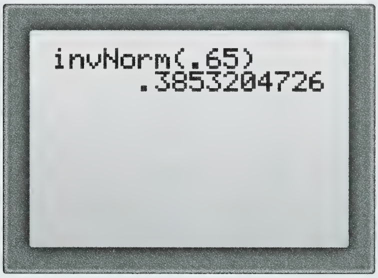


If 35% of the students obtained a higher mark than Djon, 65% of the students received a mark less than or equal to Djon's mark.

Determine the z-score corresponding to x.

Method 1: Using a Graphing Calculator

2nd [DISTR] 3 (3:invNorm()
• 6 5) ENTER



```
invNorm(.65)
         .3853204726
```

$\therefore\ z \doteq 0.39$

Method 2: Using Tables

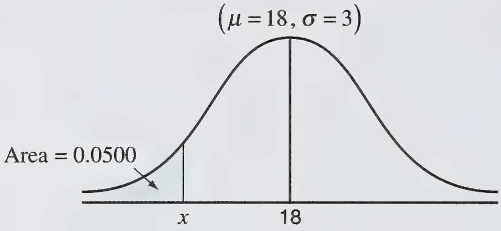
Look for an area closest to 0.6500 in the z-score tables. The closest value in the body of the tables is 0.6517, which corresponds to a z-score of 0.39.

Use the formula $z = \frac{x-\mu}{\sigma}$ to find x.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ 0.39 &= \frac{x-63}{11} \\ 0.39(11) &= x-63 \\ 4.29 &= x-63 \\ 4.29+63 &= x \\ x &= 67.29 \end{aligned}$$

Djon's mark was approximately 67.

7. Sketch the distribution. Let x be the warranty period.



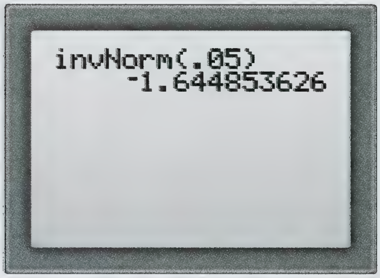
Determine the z-score corresponding to x.

Method 1: Using a Graphing Calculator

2nd [DISTR] 3 (3:invNorm()

• 0 5) ENTER

$\therefore\ z \doteq -1.64$



```
invNorm(.05)
        -1.644853626
```

Method 2: Using Tables

Look for an area closest to 0.0500 in the z-score tables. You will notice there are two values that are closest to 0.0500: 0.0495, which corresponds to a z-score of -1.65, and 0.0505, which corresponds to a z-score of -1.64. In this case, use the midpoint of these two z-scores.

$$\begin{aligned}\therefore\ z &= \frac{-1.65 + (-1.64)}{2} \\ &= -1.645\end{aligned}$$

Use the formula $z = \frac{x-\mu}{\sigma}$ to find x.

Method 1

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ -1.64 &\doteq \frac{x-18}{3} \\ -1.64(3) &\doteq x-18 \\ -4.92 &\doteq x-18 \\ x &\doteq 13.08 \end{aligned}$$

Method 2

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ -1.645 &\doteq \frac{x-18}{3} \\ -1.645(3) &\doteq x-18 \\ -4.935 &\doteq x-18 \\ x &\doteq 13.065 \end{aligned}$$

The warranty period should be about 13 months.

8. Textbook exercise "Communicating the Ideas," p. 125

Answers will vary. A sample answer is given.

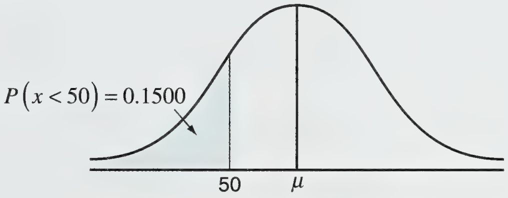
Example

Lynn obtained a mark of 50 on her math exam. She was told her mark was better than 15% of the other students who wrote this test. If the marks were normally distributed and the standard deviation was 9, what was the average mark on the math test?

Solution

Sketch the distribution.

Determine the z-score corresponding to a mark of 50. From the tables, $z \doteq -1.04$.



Use $z = \frac{x-\mu}{\sigma}$ to determine μ.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ -1.04 &\doteq \frac{50-\mu}{9} \\ -1.04(9) &\doteq 50-\mu \\ -9.36 &\doteq 50-\mu \\ \mu &\doteq 59.36 \end{aligned}$$

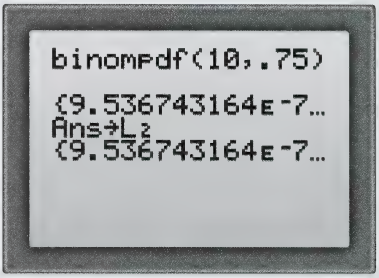
The average mark on the math test was approximately 59.

Activity 5: The Normal Approximation to a Binomial Distribution

1. **Textbook exercises 1, 2, and 3 of "Investigation: Calculating Probabilities," pp. 126 and 127**

   1. a. Press the following to generate the binomial distribution.

   2nd [DISTR] 0 (0:binompdf() 1 0 , • 7 5) ENTER

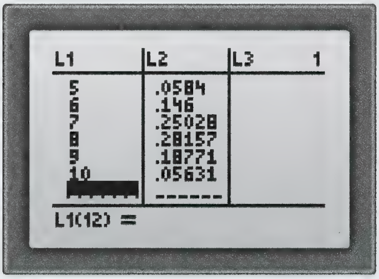


```
binompdf(10,.75)
{9.536743164E-7…
Ans→L2
{9.536743164E-7…
```

Then store the probabilities in list L2.

STO➧ 2nd [L2] ENTER

Press STAT 1 (1:Edit…) and enter the values 0 to 10 into list L1. The distribution should look like the following.



```
L1       L2       L3      1
5        .0584
6        .146
7        .25028
8        .28157
9        .18771
10       .05631
         ------
L1(12) =
```

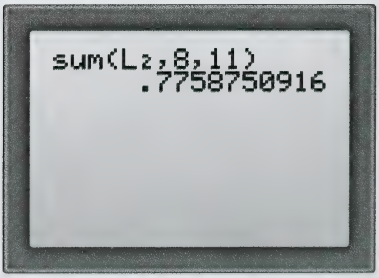
To determine the probability of making at least 7 free throws, you must find the sum of the probability of making exactly 7 free throws, exactly 8 free throws, exactly 9 free throws, and exactly 10 free throws. This can be calculated in two ways:

Method 1: Using a Graphing Calculator

Use the sum feature on your graphing calculator.

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L2] , 8 , 1 1) ENTER



```
sum(L2,8,11)
          .7758750916
```

Note: Notice that making 7 free throws corresponds to the 8th item in the list and that making 10 free throws corresponds to the 11th item in the list. This is because the list starts with making 0 free throws, not making 1 free throw. Keep this in mind when using this feature.

Method 2: Summing the Probabilities from L2

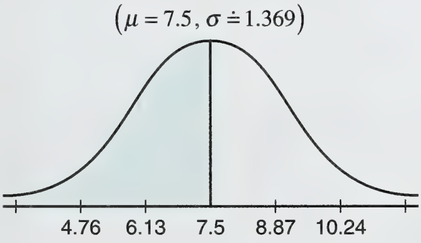
$$\begin{aligned} P(\text{at least 7 FTs}) &= P(\text{exactly 7 FTs}) + P(\text{exactly 8 FTs}) + P(\text{exactly 9 FTs}) \\ &\quad + P(\text{exactly 10 FTs}) \\ &\doteq 0.250\,28 + 0.281\,57 + 0.187\,71 + 0.056\,31 \\ &\doteq 0.775\,87 \end{aligned}$$

The probability that the basketball player will make at least 7 free throws is about 0.775 87.

b. i.

$$\begin{aligned} \mu &= n \times p \\ &= 10 \times 0.75 \\ &= 7.5 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{10 \times 0.75 \times (1-0.75)} \\ &= \sqrt{10 \times 0.75 \times 0.25} \\ &\doteq 1.369 \end{aligned}$$

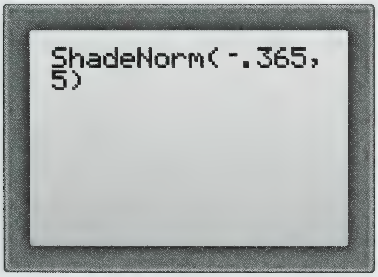
Sketch the normal distribution with a mean of 7.5 and a standard deviation of 1.369. Draw a sketch to help visualize the problem.



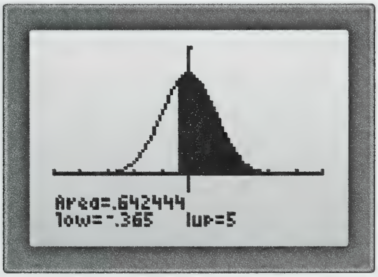
ii.

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &\doteq \frac{7-7.5}{1.369} \\ &\doteq -0.365 \end{aligned}$$

iii.



```
ShadeNorm(⁻.365,
5)
```

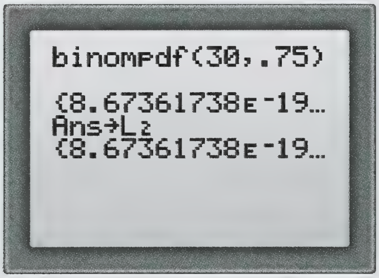


```
Area=.642444
low=⁻.365    up=5
```

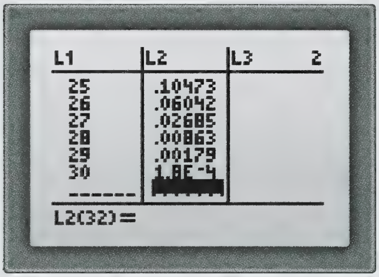
This result is about 0.13 less than the probability calculated in exercise 1.a.

c. $$\begin{aligned} n \times p \times (1-p) &= 10 \times 0.75 \times (1-0.75) \\ &= 10 \times 0.75 \times 0.25 \\ &= 1.875 \end{aligned}$$

2. a. Enter the values 0 to 30 in list L2. Then generate the distribution required and store it in list L2.



```
binompdf(30,.75)
{8.67361738ᴇ-19…
Ans→L₂
{8.67361738ᴇ-19…
```



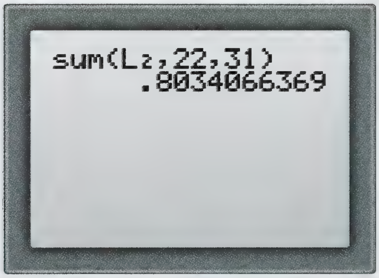
```
L1    |L2     |L3    2
25    |.10473 |
26    |.06042 |
27    |.02685 |
28    |.00863 |
29    |.00179 |
30    |1.8E-4 |
------|       |
L2(32)=
```

Determine the sum of making exactly 21 free throws, exactly 22 free throws, and so on to exactly 30 free throws.

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L2]

, 2 2 , 3 1) ENTER



```
sum(L₂,22,31)
          .8034066369
```

Remember: Because the list starts with making 0 free throws, making exactly 21 free throws corresponds to the 22nd item in the list. Likewise, making exactly 30 free throws corresponds to the 31st item in the list.

Note: You can also determine the sum by adding the individual probabilities yourself; but using the Sum feature is much quicker with less chance for error.

Therefore, the probability of making at least 21 free throws out of 30 is 0.803 41.

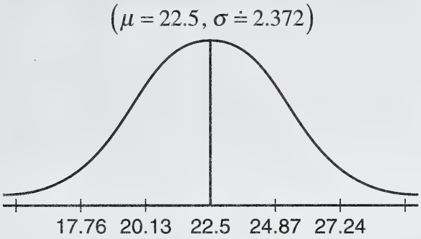
b. i. $\mu = n \times p$

$= 30 \times 0.75$

$= 22.5$

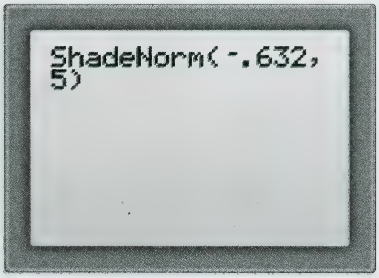
$$\begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{30 \times 0.75 \times (1-0.75)} \\ &= \sqrt{30 \times 0.75 \times 0.25} \\ &\doteq 2.371\,708\,245\end{aligned}$$

Sketch the normal distribution with a mean of 22.5 and a standard deviation of about 2.372.

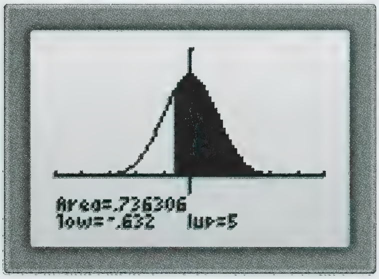


ii. $$\begin{aligned}z &= \frac{x-\mu}{\sigma} \\ &\doteq \frac{21-22.5}{2.372} \\ &\doteq -0.632\end{aligned}$$

iii.



```
ShadeNorm(⁻.632,
5)
```



```
Area=.736306
low=⁻.632    up=5
```

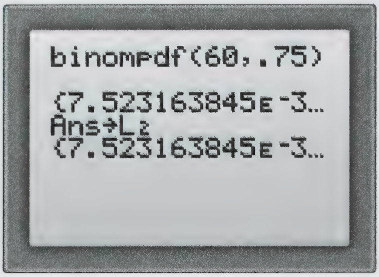
This result is about 0.067 less than the probability calculated in exercise 2.a.

c. $n \times p \times (1-p) = 30 \times 0.75 \times 0.25$

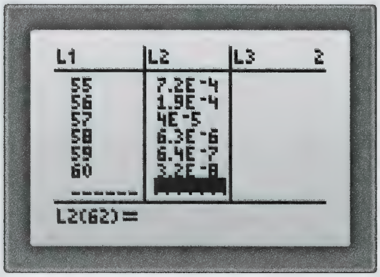
$= 5.625$

Activity 5 (continued)

3. **a.** Enter the values 0 to 60 in list L1. Then generate the distribution required, and store it in list L2.



```
binompdf(60,.75)
{7.523163845e-3…
Ans→L2
{7.523163845e-3…
```



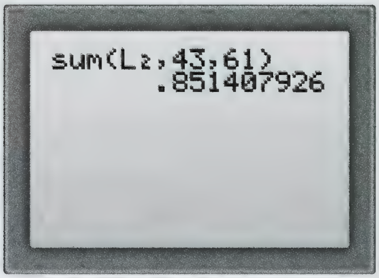
```
L1      L2      L3     2
 55     7.2E-4
 56     1.9E-4
 57     4E-5
 58     6.3E-6
 59     6.4E-7
 60     3.2E-8
 ------
L2(62)=
```

Determine the sum of making exactly 42 free throws, exactly 43 free throws, and so on to exactly 60 free throws.

Select the MATH menu.

2nd [LIST] ◀ 5 (5:sum() 2nd [L2]

, 4 3 , 6 1) ENTER



```
sum(L2,43,61)
          .851407926
```

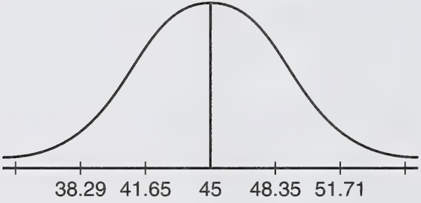
b. **i.**

$$\mu = n \times p$$
$$= 60 \times 0.75$$
$$= 45$$

$$\sigma = \sqrt{n \times p \times (1 - p)}$$
$$= \sqrt{60 \times 0.75 \times 0.25}$$
$$\doteq 3.354$$

Sketch the normal distribution with a mean of 45 and a standard deviation of about 3.354.

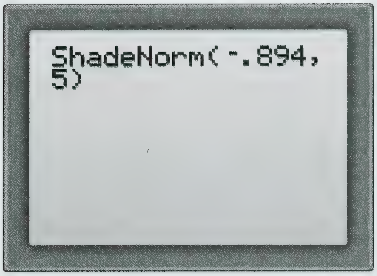
$(\mu = 45, \sigma \doteq 3.354)$



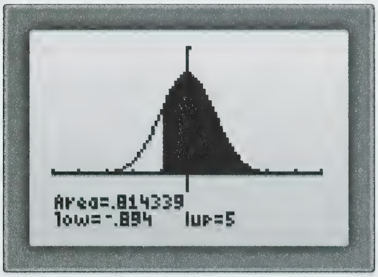
ii.

$$z = \frac{x - \mu}{\sigma}$$
$$\doteq \frac{42 - 45}{3.354}$$
$$\doteq -0.894$$

iii.



```
ShadeNorm(⁻.894,
5)
```



```
Area=.814339
low=⁻.894    up=5
```

This result is about 0.04 less than the probability calculated in exercise 3.a.

c. $$\begin{aligned} n\times p\times(1-p) &= 60\times 0.75\times(1-0.75) \\ &= 60\times 0.75\times 0.25 \\ &= 11.25 \end{aligned}$$

2. **a.** Two reasons why a normal approximation to the binomial distribution should not be used if either $n\times p$ or $n\times(1-p)$ is less than 5 are as follows:

Reason 1: If there are only a few trials in the experiment, a marked difference can occur between the results of the experiment and the theoretical distribution of the results. If *n* is small, $n\times p$ or $n\times(1-p)$ will be less than 5.

Reason 2: The normal distribution is a continuous curve. If the number of trials in the binomial experiment is small, the histogram of the distribution of the outcomes of the experiment, which consists of discrete or separate bars, will not match the normal curve closely enough. It is only when the number of trials is large and $n\times p\geq 5$ and $n\times(1-p)\geq 5$ that the histogram approaches the slope of the normal curve.

b. Some factors that affect how close an estimate of the probability is to the actual probability are as follows:

- the value of *n* (number of trials in the binomial experiment)

  The larger *n* is, the closer the estimate will be to the actual probability.

- the match between the normal curve and the histogram for the binomial experiment

  The closer the slope of the histogram is to the normal curve, the better the estimates of the probabilities will be.

- the values of $n\times p$ and $n\times(1-p)$

  The larger the values of $n\times p$ and $n\times(1-p)$, the better the approximations will be. Because these products depend on *p*, the probability of success on any given trial, this variable will affect the estimates.

3. Textbook exercises 4, 5, and 6 of "Exercises: Checking Your Skills," pp. 130 and 131

4. With a fair 6-sided die, the probability of a 1 is $\frac{1}{6} = 0.1\overline{6}$.

$$n \times p = 1000 \times \frac{1}{6} = 166.\overline{6} \qquad n \times (1-p) = 1000 \times \frac{5}{6} = 833.\overline{3}$$

Because $n \times p \geq 5$ and $n \times (1-p) \geq 5$, it is appropriate to use the normal approximation to this binomial distribution.

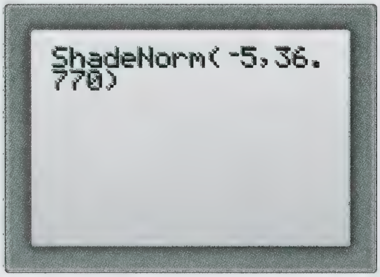
Calculate the mean and the standard deviation.

$$\begin{aligned}\mu &= n \times p \\ &= 1000 \times \frac{1}{6} \\ &= 166\frac{2}{3}\end{aligned} \qquad \begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{1000 \times \frac{1}{6} \times \left(1 - \frac{1}{6}\right)} \\ &= \sqrt{1000 \times \frac{1}{6} \times \frac{5}{6}} \\ &\doteq 11.785\end{aligned}$$

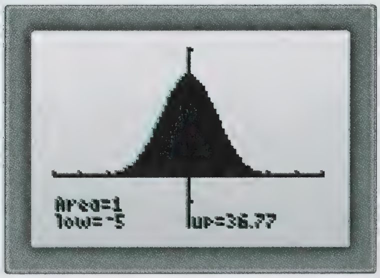
Calculate the z-score for $x = 600$.

$$\begin{aligned}z &= \frac{x - \mu}{\sigma} \\ &\doteq \frac{600 - 166\frac{2}{3}}{11.785} \\ &\doteq 36.770\end{aligned}$$

Use your graphing calculator to determine the probability of at most 600 ones.



ShadeNorm(-5,36.770)



Area=1
low=-5
up=36.77

The probability of at most 600 ones is approximately 1.

5. Because 12% of consumers choose a particular brand of pain reliever, when consumers are selected at random, the probability that any one of them would choose this brand is $p = 0.12$. Because 325 consumers are selected, $n = 325$.

Determine the values of $n \times p$ and $n \times (1 - p)$ to check if it is appropriate to use the normal approximation.

$$\begin{aligned} n \times p &= 325 \times 0.12 \\ &= 39 \end{aligned} \qquad \begin{aligned} n \times (1 - p) &= 325 \times (1 - 0.12) \\ &= 286 \end{aligned}$$

Since $n \times p \geq 5$ and $n \times (1 - p) \geq 5$, it is appropriate to use the normal approximation.

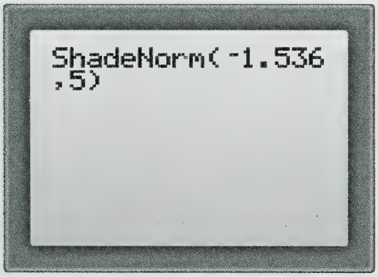
Next, determine the mean and the standard deviation.

$$\begin{aligned} \mu &= n \times p \\ &= 325 \times 0.12 \\ &= 39 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{n \times p \times (1 - p)} \\ &= \sqrt{325 \times 0.12 \times (1 - 0.12)} \\ &\doteq 5.858 \end{aligned}$$

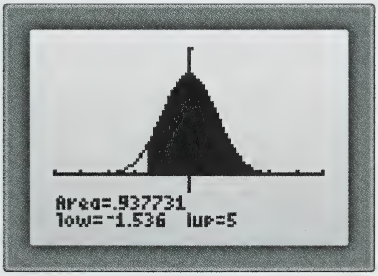
You are interested in the probability that at least 30 consumers purchase this brand. Therefore, determine the z-score for $x = 30$.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &\doteq \frac{30 - 39}{5.858} \\ &\doteq -1.536 \end{aligned}$$

Use your graphing calculator to determine the probability that at least 30 of the consumers surveyed purchase this brand.



ShadeNorm(-1.536
,5)



Area=.937731
low=-1.536 up=5

$\therefore\ P(z \geq -1.54) \doteq 0.9377$

The probability that at least 30 of the consumers surveyed purchase this brand is approximately 0.9377.

6. Since 46% of Canadians have type O blood, the probability that a person selected with this blood type is $p = 0.46$. Since 500 volunteers donate blood, $n = 500$.

Determine the values of $n \times p$ and $n \times (1 - p)$ to check if it is appropriate to use the normal approximation.

$$\begin{aligned} n \times p &= 500 \times 0.46 \\ &= 230 \end{aligned} \qquad \begin{aligned} n \times (1 - p) &= 500 \times (1 - 0.46) \\ &= 270 \end{aligned}$$

Since $n \times p \geq 5$ and $n \times (1 - p) \geq 5$, it is appropriate to use the normal approximation.

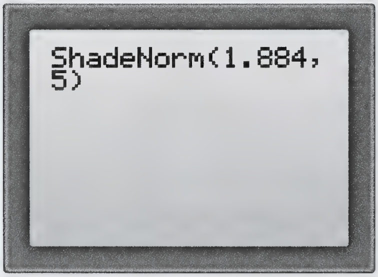
Determine the mean and the standard deviation.

$$\begin{aligned} \mu &= n \times p \\ &= 500 \times 0.46 \\ &= 230 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{n \times p \times (1 - p)} \\ &= \sqrt{500 \times 0.46 \times (1 - 0.46)} \\ &\doteq 11.145 \end{aligned}$$

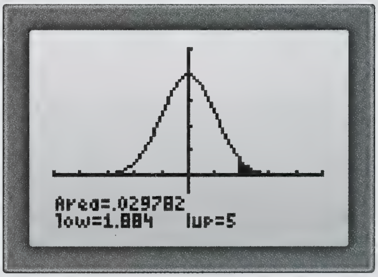
You are interested in the probability that more than half have type O blood; therefore, $x = 251$. Determine the z-score for $x = 251$.

$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &\doteq \frac{251 - 230}{11.145} \\ &\doteq 1.884 \end{aligned}$$

Use your graphing calculator to determine the probability that more than half of the volunteers have type O blood.



```
ShadeNorm(1.884,
5)
```



```
Area=.029782
low=1.884    up=5
```

$\therefore\ P(z \geq 1.89) \doteq 0.0298$

The probability that more than half have type O blood is approximately 0.0298.

4. **Textbook exercise "Communicating the Ideas," p. 131**

Answers will vary. A sample answer is given.

A normal distribution describes continuous data; a binomial distribution describes discrete data.

For a normal distribution, there are an infinite number of data points; for a binomial distribution, there are a finite number of points.

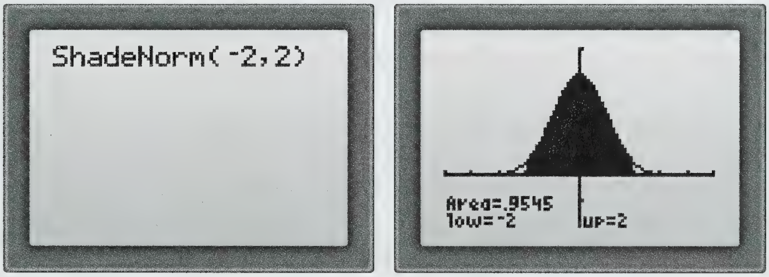
A normal distribution describes how characteristics are distributed in a population; a binomial distribution describes the relative frequencies for outcomes of a binomial experiment.

A normal distribution can be used to approximate a binomial distribution if $n \times p \geq 5$ and $n \times (1-p) \geq 5$, where n is the number of trials in the binomial distribution and p is the probability of success of any given trial.

Activity 6: Confidence Intervals

1. **Textbook exercises 1, 2, and 3 of "Investigation: Find the Symmetric Interval Under the Standard Normal Curve That Contains 95% of the Data," p. 135**

1.

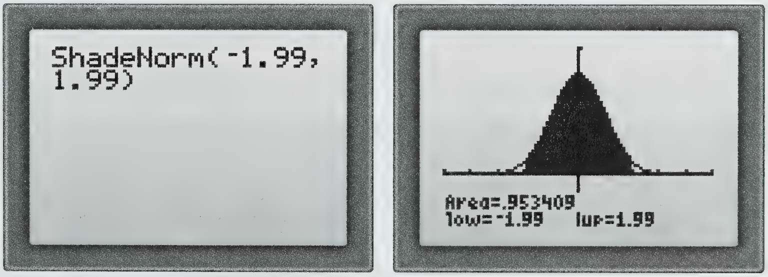


```
ShadeNorm(-2,2)
```

```
Area=.9545
low=-2        up=2
```

The area between $z = -2$ and $z = 2$ is approximately 0.9545. This is approximately 95.45% of the data.

2.

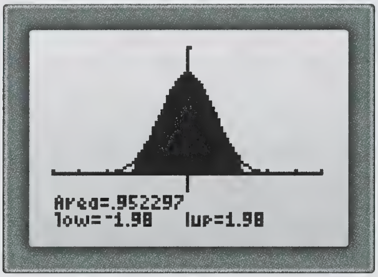


```
ShadeNorm(-1.99,
1.99)
```

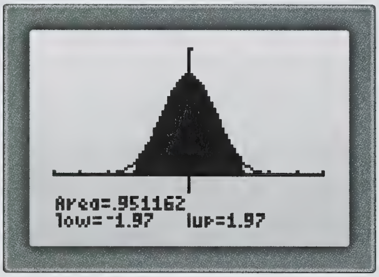
```
Area=.953409
low=-1.99    up=1.99
```

The area between $z = -1.99$ and $z = 1.99$ is approximately 0.9534.

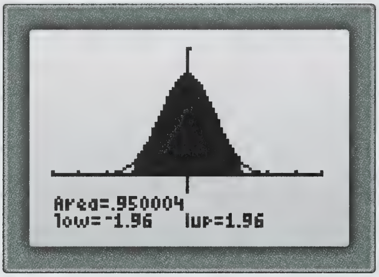
3. The following calculator screens show the results of several more slight changes in the boundaries.



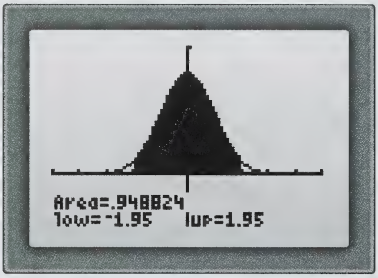
```
Area=.952297
low=-1.98   up=1.98
```



```
Area=.951162
low=-1.97   up=1.97
```



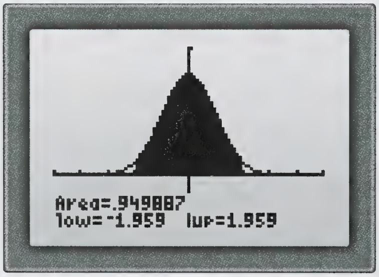
```
Area=.950004
low=-1.96   up=1.96
```



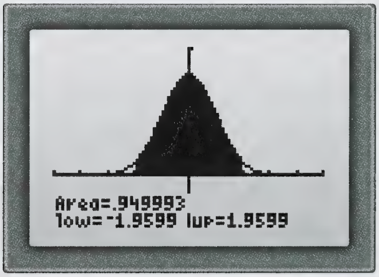
```
Area=.948824
low=-1.95   up=1.95
```

a. The area between $z=-1.96$ and $z=1.96$ was extremely close to 0.95.

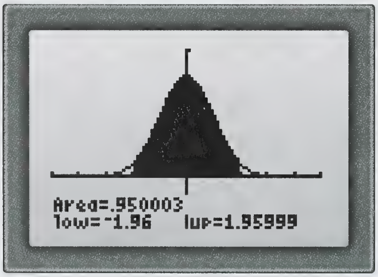
b. The area is not exactly 0.95 because the boundaries are a little bit too large. You could find a closer approximation by calculating the area using 0.001 steps, then 0.0001 steps, and so on. The following calculator screens show this movement toward a better value.



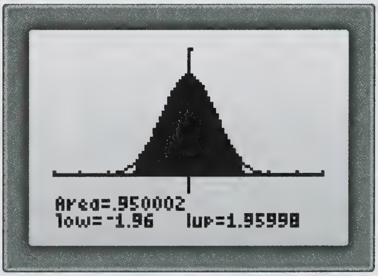
```
Area=.949887
low=-1.959  up=1.959
```



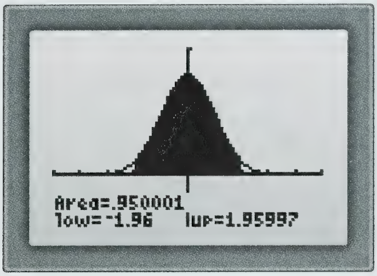
```
Area=.949993
low=-1.9599 up=1.9599
```



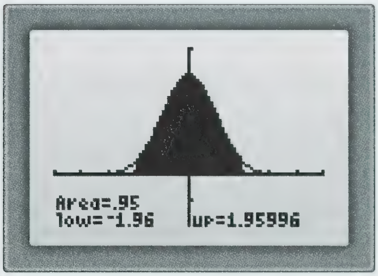
```
Area=.950003
low=-1.96   up=1.95999
```



```
Area=.950002
low=-1.96   up=1.95998
```



```
Area=.950001
low=-1.96    up=1.95997
```



```
Area=.95
low=-1.96    up=1.95996
```

The intervals here are symmetrical. The calculator rounded the lower bound to -1.96 because it ran out of room to display the entire number.

c. Ninety-five percent of the data is located within 1.959 96 standard deviations of the mean. For practical purposes, round this value to 1.96 standard deviations.

2. Textbook exercise 1 of "Discussing the Ideas," p. 138

1. a. A 95% confidence interval is $\mu \pm 1.96\sigma$. It is the range of scores that contain 95% of the data in the distribution.

A 90% confidence interval is $\mu \pm 1.64\sigma$. It is the range of scores that contain 90% of the data in the distribution.

A 99% confidence interval is $\mu \pm 2.58\sigma$. It is the range of scores that contain 99% of the data in the distribution.

b. The range of scores is least for the 90% confidence interval and greatest for the 99% confidence interval. A 95% confidence interval is a compromise between the other two and is the most often used interval in reporting survey results.

3. a. Textbook exercises 2 and 3 of "Discussing the Ideas," p. 138

2. When reporting survey results, it is important to give the margin of error. The survey cannot predict exactly what the opinion of the population is. The survey's audience should be aware of the measurement error and the range of values within which the population is likely to be. It is a question of being honest when results are reported.

3. As the sample size increases, the margin of error decreases. The more people you survey, the more confident you will be in your results. If you could survey the entire population, your margin of error would be reduced to 0%.

b. Textbook exercises 1.a., 1.b., 2, 5, and 7 of "Exercises: Checking Your Skills," pp. 139 to 141

1. **a.** The 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma \\ &= 68.08 - 1.96(4.2) \\ &\doteq 59.848\end{aligned}$$

$$\begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma \\ &= 68.08 + 1.96(4.2) \\ &\doteq 76.312\end{aligned}$$

The 95% confidence interval is approximately between 59.848 and 76.312.

$$\begin{aligned}\text{Percent margin of error} &= \pm\frac{1.96\sigma}{n}\times 100\% \\ &= \pm\frac{1.96(4.2)}{92}\times 100\% \\ &\doteq \pm 8.948\%\end{aligned}$$

b. The 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma \\ &= 102.446 - 1.96(6.67) \\ &\doteq 89.373\end{aligned}$$

$$\begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma \\ &= 102.446 + 1.96(6.67) \\ &\doteq 115.519\end{aligned}$$

The 95% confidence interval is approximately between 89.373 and 115.519.

$$\begin{aligned}\text{Percent margin of error} &= \pm\frac{1.96\sigma}{n}\times 100\% \\ &= \pm\frac{1.96(6.67)}{181}\times 100\% \\ &\doteq \pm 7.223\%\end{aligned}$$

2. **a.** The 66 teenagers who watch *The National* out of the 150 surveyed is 44% of the sample. Therefore, $p = 0.44$.

Use the value of p to determine the mean and standard deviation.

$$\begin{aligned}\mu &= n \times p \\ &= 150 \times 0.44 \\ &= 66\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{150 \times 0.44 \times (1-0.44)} \\ &\doteq 6.079\end{aligned}$$

The 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma \\ &\doteq 66 - 1.96(6.079) \\ &\doteq 54\end{aligned} \qquad \begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma \\ &= 66 + 1.96(6.079) \\ &\doteq 78\end{aligned}$$

The 95% confidence interval is approximately between 54 and 78.

b. $$\begin{aligned}\text{Percent margin of error} &= \pm\frac{1.96\sigma}{n} \times 100\% \\ &= \pm\frac{1.96(6.079)}{150} \times 100\% \\ &\doteq \pm 7.9\%\end{aligned}$$

5. Determine the mean and standard deviation.

$n = 200$ and $p = 0.10$

$$\begin{aligned}\therefore\ \mu &= n \times p \\ &= 200 \times 0.10 \\ &= 20\end{aligned} \qquad \begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)} \\ &= \sqrt{200 \times 0.10 \times (1-0.10)} \\ &\doteq 4.243\end{aligned}$$

The 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma \\ &\doteq 20 - 1.96(4.243) \\ &\doteq 12\end{aligned} \qquad \begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma \\ &\doteq 20 + 1.96(4.243) \\ &\doteq 28\end{aligned}$$

The 95% confidence interval is approximately between 12 and 28.

$$\begin{aligned}\text{Percent margin of error} &= \pm\frac{1.96\sigma}{n} \times 100\% \\ &\doteq \pm\frac{1.96(4.243)}{200} \times 100\% \\ &\doteq \pm 4.2\%\end{aligned}$$

7. **a.** There were 2752 respondents who judged the painkiller effective out of the 3000 users surveyed.

$$\begin{aligned}\therefore\ p &= \frac{2752}{3000}\\ &\doteq 0.9173\end{aligned}$$

Determine the mean and standard deviation.

$$\begin{aligned}\mu &= n \times p\\ &\doteq 3000 \times 0.9173\\ &\doteq 2752\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{n \times p \times (1-p)}\\ &\doteq \sqrt{3000 \times 0.9173 \times (1-0.9173)}\\ &\doteq 15.086\end{aligned}$$

The 95% confidence interval is $\mu \pm 1.96\sigma$.

$$\begin{aligned}\text{Lower bound} &= \mu - 1.96\sigma\\ &\doteq 2752 - 1.96(15.086)\\ &\doteq 2722\end{aligned}$$

$$\begin{aligned}\text{Upper bound} &= \mu + 1.96\sigma\\ &\doteq 2752 + 1.96(15.086)\\ &\doteq 2782\end{aligned}$$

The 95% confidence interval is approximately between 2722 and 2782.

b. Determine the margin of error for the improved pain reliever.

$$\begin{aligned}\text{Percent margin of error} &= \pm\frac{1.96\sigma}{n} \times 100\%\\ &\doteq \pm\frac{1.96(15.086)}{3000} \times 100\%\\ &\doteq \pm 0.99\%\end{aligned}$$

With 95% confidence you know that $91.73 \pm 1.0\%$, or between 90.73% and 92.73%, prefer the improved painkiller.

Determine the margin of error for the regular painkiller for which $p = 0.90$.

$$\begin{aligned}\text{Percent margin of error} &= \pm 1.96\sqrt{\frac{p(1-p)}{n}} \times 100\%\\ &= \pm 1.96\sqrt{\frac{0.90(1-0.90)}{3000}} \times 100\%\\ &\doteq \pm 1.1\%\end{aligned}$$

With 90% confidence you know that $90 \pm 1.1\%$, or between 88.9% and 91.1% of respondents, find the regular painkiller effective.

Because the confidence intervals overlap for the two painkillers, the differences in the surveyed results are within the margin of error. The survey doesn't tell the company if the extra-strength painkiller is more effective than the regular-strength painkiller. Therefore, the company should not replace its regular painkiller based on these survey results. Companies have reasons other than effectiveness to introduce new or improved products. These could include patents running out, advertising concerns, or the cost of manufacturing.

c. **Textbook exercise 10 of "Exercises: Extending Your Thinking," p. 141**

10. a. Since the range 3% to 9% can be expressed as $6\% \pm 3\%$, the margin of error is $\pm 3\%$.

Note: $p = \frac{12}{200} = 0.06$ or 6%

b. $$\text{Margin of error} = \pm 1.96\sqrt{\frac{p(1-p)}{n}}$$

$$0.015 = \pm 1.96\sqrt{\frac{0.06(1-0.06)}{n}}$$

$$\frac{0.015}{1.96} = \pm\sqrt{\frac{0.06(0.94)}{n}}$$

$$\left(\frac{0.015}{1.96}\right)^2 = \left(\pm\sqrt{\frac{0.06(0.94)}{n}}\right)^2$$

$$\left(\frac{0.015}{1.96}\right)^2 = \frac{0.06(0.94)}{n}$$

$$\left(\frac{0.015}{1.96}\right)^2 n = 0.06(0.94)$$

$$n = \frac{0.06(0.94)}{\left(\frac{0.015}{1.96}\right)^2}$$

$$\doteq 963$$

About 963 people would have to be sampled to reduce the margin of error to $\pm 1.5\%$.

4. **Textbook exercise "Communicating the Ideas," p. 141**

The confidence interval is the range of values in the distribution within which a stated percentage of the data lies. For a 95% confidence interval, 95% of the data would lie within this range.

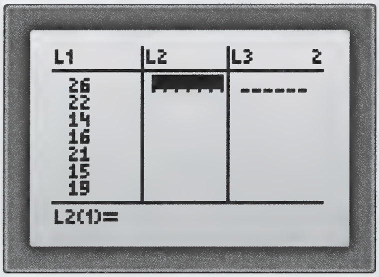
The margin of error is the percentage added and subtracted from the mean of the sample within which the stated percentage lies.

Confidence intervals and margins of error are useful when you interpret survey results because you want to determine the range of values within which your survey results likely occur. Without these measures, you cannot make appropriate decisions or predictions based on your results.

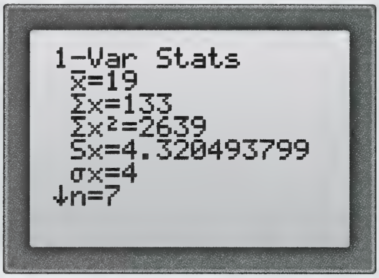
Module Review

1. **a.** Use your graphing calculator to determine the mean and standard deviation. Enter the data from Set A as list L1. Press STAT, and select the CALC menu; then press 1 (1:1–Var Stats) ENTER.

   Note: Clear any lists before entering Set A.



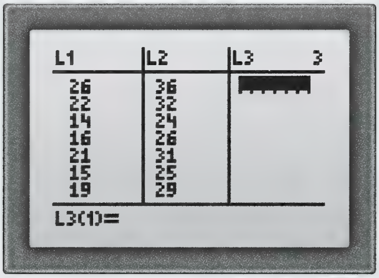
```
L1      |L2      |L3     2
 26     |        |------
 22     |        |
 14     |        |
 16     |        |
 21     |        |
 15     |        |
 19     |        |
L2(1)=
```



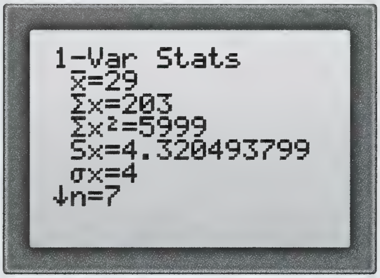
```
1-Var Stats
 x̄=19
 Σx=133
 Σx²=2639
 Sx=4.320493799
 σx=4
↓n=7
```

The mean is 19, and the standard deviation is 4.

b. Enter the data in list L2. Press STAT, and select the CALC menu; then press 1 2nd [L2] ENTER.



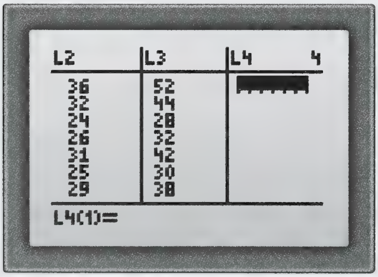
```
L1      |L2      |L3     3
 26     | 36     |
 22     | 32     |
 14     | 24     |
 16     | 26     |
 21     | 31     |
 15     | 25     |
 19     | 29     |
L3(1)=
```



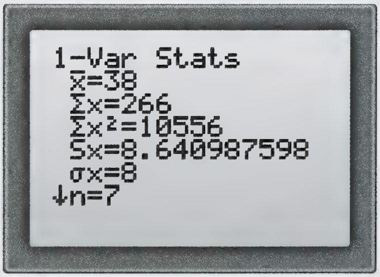
```
1-Var Stats
 x̄=29
 Σx=203
 Σx²=5999
 Sx=4.320493799
 σx=4
↓n=7
```

The mean is 29, and the standard deviation is 4.

c.



```
L2      |L3      |L4     4
 36     | 52     |
 32     | 44     |
 24     | 28     |
 26     | 32     |
 31     | 42     |
 25     | 30     |
 29     | 38     |
L4(1)=
```



```
1-Var Stats
 x̄=38
 Σx=266
 Σx²=10556
 Sx=8.640987598
 σx=8
↓n=7
```

The mean is 38, and the standard deviation is 8.

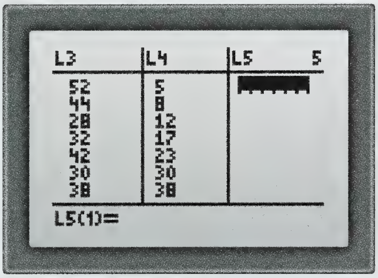
d. Data set B was obtained by adding 10 to each value in data set A. The mean of data set B is 10 more than the mean of data set A, but the standard deviations are the same.

e. Data set C was obtained by multiplying each value in data set A by 2. The mean and standard deviation of set C are each 2 times larger than the mean and standard deviation of set A.

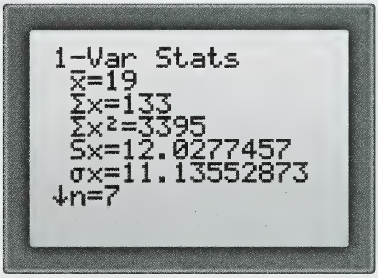
f. If each value of a data set is changed by adding *k*, the mean of the new data set is *k* more than the mean of the original set and the standard deviation remains the same.

If each value of a data set is multiplied by a positive factor, *f*, the mean and the standard deviation of the new set is *f* times the mean and standard deviation of the original set.

g. **i.** Repeat the steps outlined in question a. to find the mean and standard deviation for set D.



```
L3     L4     L5     5
52     5
44     8
28     12
32     17
42     23
30     30
38     38
L5(1)=
```



```
1-Var Stats
 x̄=19
 Σx=133
 Σx²=3395
 Sx=12.0277457
 σx=11.13552873
↓n=7
```

The mean is 19, and the standard deviation is approximately 11.136.

ii. If you triple the elements in data set D, you will obtain the following set:

$$E=\{15, 24, 36, 51, 69, 90, 114\}$$

The mean will be $19\times3=57$, and the standard deviation will be $11.136\times3\doteq33.408$.

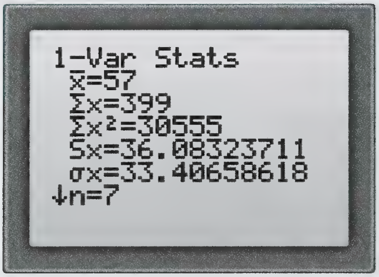
iii. If you add 5 to each of the elements in set D, you will obtain the following set:

$$F=\{10, 13, 17, 22, 28, 35, 43\}$$

The mean will be $19+5=24$, and the standard deviation will remain the same, approximately 11.136.

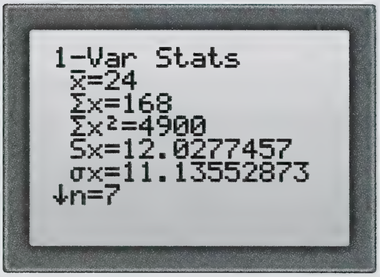
iv.

Data Set E



```
1-Var Stats
 x̄=57
 Σx=399
 Σx²=30555
 Sx=36.08323711
 σx=33.40658618
↓n=7
```

Data Set F



```
1-Var Stats
 x̄=24
 Σx=168
 Σx²=4900
 Sx=12.0277457
 σx=11.13552873
↓n=7
```

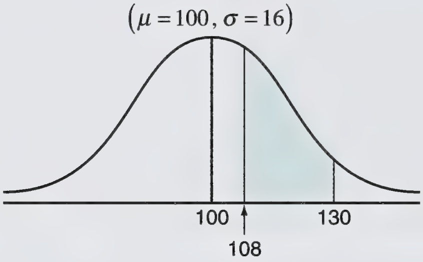
The mean of set E is 57, and its standard deviation is approximately 33.407.

The mean of set F is 24, and its standard deviation is approximately 11.136.

These values closely match the values predicted.

2. **Textbook exercises 5, 6, and 8 of Part B of "What Should I Be Able to Do?," p. 146**

   5. Sketch the distribution.



Use the formula $z = \frac{x-\mu}{\sigma}$ to convert $x = 108$ and $x = 130$ into z-scores.

For $x = 108$,

$$z = \frac{x-\mu}{\sigma}$$
$$z_{108} = \frac{108-100}{16}$$
$$= 0.50$$

For $x = 130$,

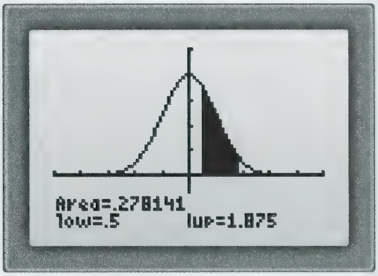
$$z = \frac{x-\mu}{\sigma}$$
$$z_{130} = \frac{130-100}{16}$$
$$= 1.875$$

Using the Shade Normal feature on your graphing calculator, find the area between the z-scores. Remember to turn all Stat Plots to OFF.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() .

5 , 1 . 8 7 5) ENTER



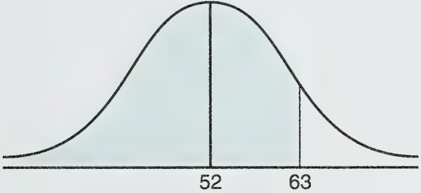
Area=.278141
low=.5 up=1.875

$\therefore\ p(108 < x < 130) \doteq 0.2781$

The probability that a randomly selected individual has an IQ between 108 and 130 is approximately 0.2781.

6. Sketch the distribution.

$(\mu = 52, \sigma = 8)$



Convert $x = 63$ into a z-score.

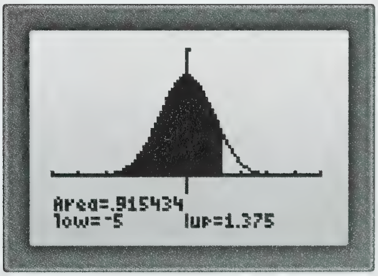
$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{63-52}{8} \\ &= 1.375 \end{aligned}$$

Use the Shade Normal feature on your graphing calculator to find the area less than $z = 1.375$.
Remember: Use -5 as the lower bound.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm() (–)

5 , 1 . 3 7 5) ENTER



Area=.915434
low=-5 up=1.375

$\therefore\ p(x < 63) \doteq 0.9154$

The percent of students that have a score less than 63 is approximately 91.54%.

8. Because 5% of the light bulbs are defective, $p = 0.05$. Also, because 1000 bulbs are selected at random for the sample, $n = 1000$.

Check if it is appropriate to use the normal approximation.

$$\begin{aligned} n \times p &= 1000 \times 0.05 \\ &= 50 \end{aligned} \qquad \begin{aligned} n \times (1 - p) &= 1000 \times (1 - 0.05) \\ &= 950 \end{aligned}$$

Since $n \times p \geq 5$ and $n \times (1 - p) \geq 5$, it is appropriate to use the normal approximation.

Next, determine the mean and the standard deviation.

$$\begin{aligned} \mu &= n \times p \\ &= 1000 \times 0.05 \\ &= 50 \end{aligned} \qquad \begin{aligned} \sigma &= \sqrt{n \times p \times (1 - p)} \\ &= \sqrt{1000 \times 0.05 \times (1 - 0.05)} \\ &\doteq 6.892 \end{aligned}$$

You are interested in the probability that 30 or more are defective. Therefore, determine the z-score for $x = 30$.

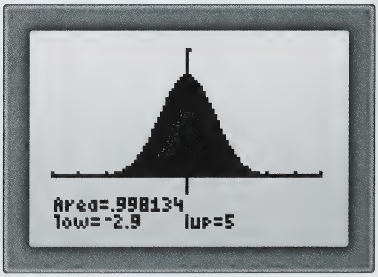
$$\begin{aligned} z &= \frac{x - \mu}{\sigma} \\ &= \frac{30 - 50}{6.89} \\ &\doteq -2.90 \end{aligned}$$

Find $p(z \geq -2.90)$ using your graphing calculator.

Select the DRAW menu.

2nd [DISTR] ▶ 1 (1:ShadeNorm(

(−) 2 . 9 , 5) ENTER



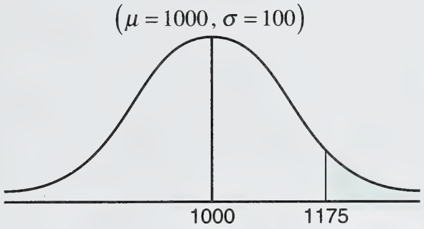
```
Area=.998134
low=⁻2.9     up=5
```

$\therefore p(x > 30) \doteq 0.9981$

The probability that at least 30 bulbs will be defective is approximately 0.9981.

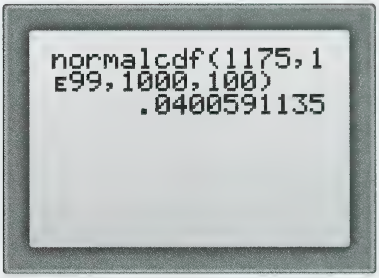
Enrichment

1. Sketch the distribution.



lower bound = 1175
upper bound = 1×10^{99}

2nd [DISTR] 2 (2:normalcdf() 1 1 7 5 ,
1 2nd [EE] 9 9 , 1 0 0 0 ,
1 0 0) ENTER

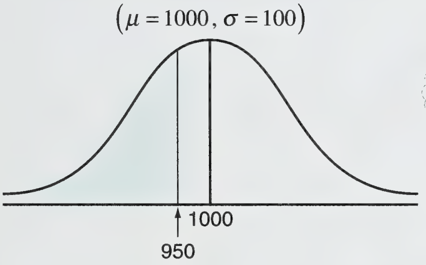


```
normalcdf(1175,1
E99,1000,100)
      .0400591135
```

$\therefore\ p(x > 1175) \doteq 0.0401$

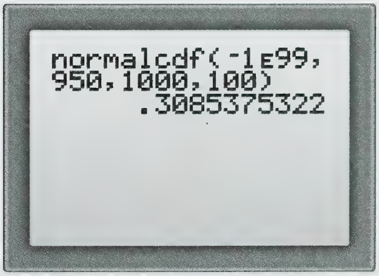
Therefore, the probability that a given light bulb will last longer than 1175 h is approximately 0.0401.

2. Sketch the distribution.



lower bound = -1×10^{99}
upper bound = 950

2nd [DISTR] 2 (2:normalcdf() (–) 1 2nd [EE]
9 9 , 9 5 0 , 1 0 0 0 ,
1 0 0 ENTER

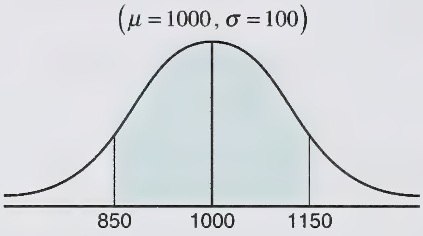


```
normalcdf(-1E99,
950,1000,100)
      .3085375322
```

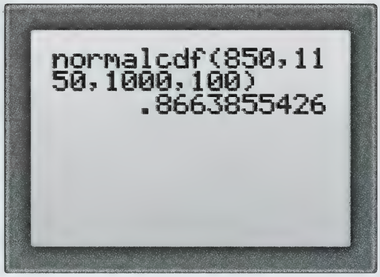
$\therefore\ p(x < 950) \doteq 0.3085$

Therefore, the probability that a given light bulb will last less than 950 h is approximately 0.3085.

3. Sketch the distribution.



2nd [DISTR] 2 (2:normalcdf() 8 5 0 , 1 1 5 0 , 1 0 0 0 , 1 0 0 , ENTER



```
normalcdf(850,11
50,1000,100)
           .8663855426
```

$\therefore\ p(850 < x < 1150) \doteq 0.8664$

Therefore, the probability that a given light bulb will last between 850 h and 1150 h is approximately 0.8664.

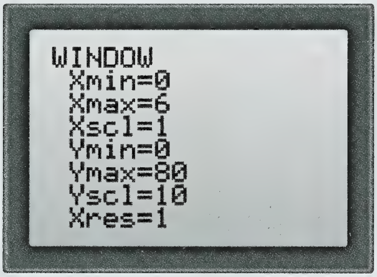
Module Project: Opinion Polls

Completing the Project

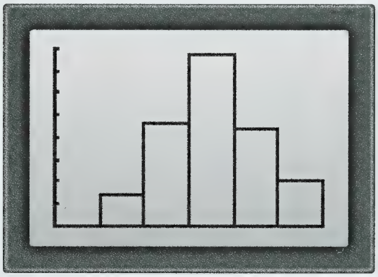
1. Textbook exercise 10 of Part C of “What Should I Be Able to Do?,” p. 147

10. The table summarizing the student’s survey results is confusing. What does the column in the table titled “Score” represent? If it is the number of respondents in each category, it should total 200. In fact, it totals 202. The calculator window showing the calculations of the mean and standard deviation should have been drawn in.

The statement that the data fits a normal distribution is unsubstantiated. If the student had entered the two numerical columns in the table as list L1 and list L2 in her graphing calculator, and had used the following graphing window, she could have displayed the data as a histogram.



```
WINDOW
 Xmin=0
 Xmax=6
 Xscl=1
 Ymin=0
 Ymax=80
 Yscl=10
 Xres=1
```



The student could then have referred to the histogram, pointing out that it appears bell-shaped and that it approximates the normal curve.

The student points out that 83% of the population would not rate the health system as excellent. In fact, the table indicates that 188 of the 202 respondents surveyed fell into this category. This is $\frac{188}{202} \doteq 93.1\%$.

Also, the percentage of the population that would rate the system as higher than “good” is quoted as 48%. Again, from the survey data summarized in the table, $14 + 46 = 60$ respondents fall in this category. This is $\frac{60}{202} \doteq 29.9\%$. The percentages 83% and 48% are incorrect based on the data.

2. Textbook exercise 11 of Part C of "What Should I Be Able to Do?," p. 148

11. The student should have summarized the survey before commenting on the results. Why was the survey conducted? Who was surveyed? How were the respondents selected? How many were surveyed in the opinion poll? What questions were asked? What were the responses? What conclusions were drawn?

Because the answers to none of these questions are given, it is impossible to comment on the numerical analysis the student presents. She begins by confusing "margin of error" with a "confidence interval." All of her calculations are either unclear or in error.

The confidence interval and margin of error should have been determined as follows:

$$n = 1071 \qquad p = 72\% \text{ or } 0.72$$

$$\begin{aligned}\mu &= n \times p\\ &= 1071 \times 0.72\\ &= 771\end{aligned}$$

$$\begin{aligned}\sigma &= \sqrt{n \times p(1-p)}\\ &= \sqrt{1071 \times 0.72(1-0.72)}\\ &= 14.69\end{aligned}$$

95% confidence interval $= \mu \pm 1.96\sigma$

$$771 - 1.96(14.69) = 742 \qquad 771 + 1.96(14.69) = 800$$

$$\frac{742}{1071} \times 100 = 69\% \qquad \frac{800}{1071} \times 100 = 75\%$$

You know that between 69% and 75% of Albertans are concerned about the health-care system with 95% confidence.

The percent margin of error is calculated as follows:

$$\begin{aligned}&\frac{\pm 1.96\sigma}{1071} \times 100\\ &= \frac{\pm 28.79}{1071} \times 100\\ &= \pm 2.7\%\end{aligned}$$

The report stated $\pm 3.0\%$ as the percent margin of error. The calculated margin of error is close to the reported margin of error.

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Welcome Page

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